UNIT - 1

# STRESSES AND STRAINS

strength of materials is a subject which deals with how solid bodies react when subjected to various types of loading. we consider solid bodies like bars loaded longitudinally, beams subjected to bending, shafts subjected to torsion, columns subjected to compression and so on.

## STRESS :-

The force of resistance offered by a body against the deformation is called the stress. The external force acting on the body is called the load. The load is applied on the body while the stress is induced in the material of the body. It is denoted by '$\sigma$' and it is expressed in $N/m^2$.

mathematically stress $(\sigma) = \dfrac{\text{External force (or) Resistive force}}{\text{cross - sectional area}}$

$$\sigma = \frac{P}{A}$$

## Types of stresses :-

stresses are divided into two types

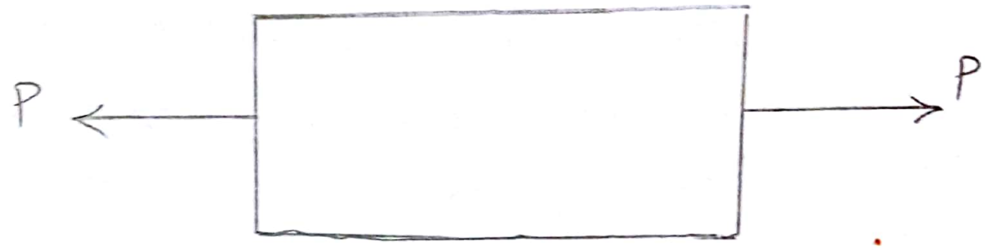
i) Normal stress

ii) Shear stress

Normal stress is the stress which acts in a direction perpendicular to the area. It is represented by '$\sigma$' (sigma). The normal stress is further divided into tensile stress and Compressive stress.

Tensile stress:-

The stress induced in a body, when subjected to two equal and opposite pull as shown in below. As a result of which there is an increase in length. is known as tensile stress.



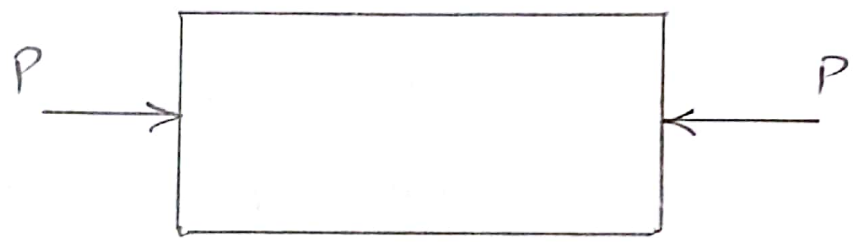

$$\text{Tensile stress } (\sigma_t) = \frac{\text{Resisting force (R)}}{\text{Cross sectional area(A)}}$$

$$\frac{\text{Pulling force}}{A} = \frac{\text{Tensile load (P)}}{A}$$

$$\sigma_t = \frac{P}{A}$$

Compressive stress :-

The stress induced in a body, when subjected to two equal and opposite pushes as shown in below. as a result of which there is a decrease in length of a body, is known as Compressive stress.

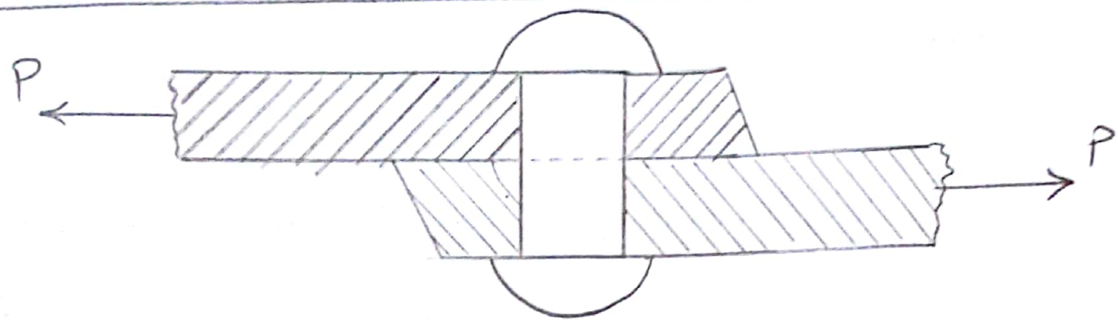


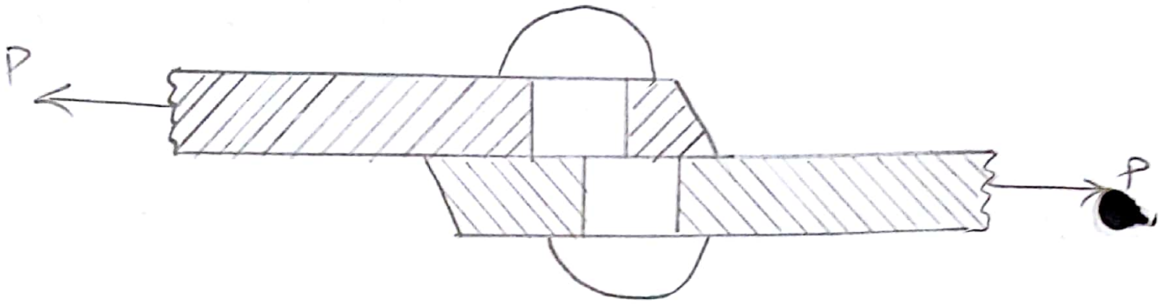

$$\text{Compressive stress } (\sigma_c) = \frac{\text{Resisting force (R)}}{\text{Area (A)}}$$

$$= \frac{\text{pushing force}}{A}$$

Shear stress:-

The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in fig. as a result of which the body tends to shear off across the section, is known as shear stress. it is denoted by '$\tau$'




$$\text{shear stress } (\tau) = \frac{\text{shear Resistance}}{\text{shear Area}}$$

$$= \frac{\text{Tangential load}}{\text{Area}}$$

## STRAIN :-

The ratio of change in dimensions of the body to the original dimensions is known as strain. strain is dimensionless.

The various types of strains are.

1) Tensile strain.
2) Compressive strain.
3) shear strain.
4) volumetric strain.

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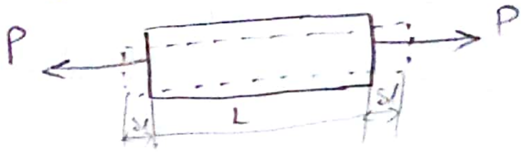
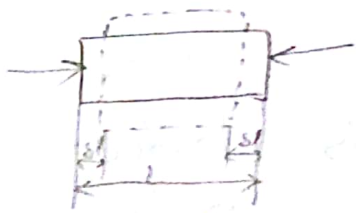
Tensile strain :-

The ratio of increase of length to the original length of the body is known as tensile strain.




Compressive strain :-

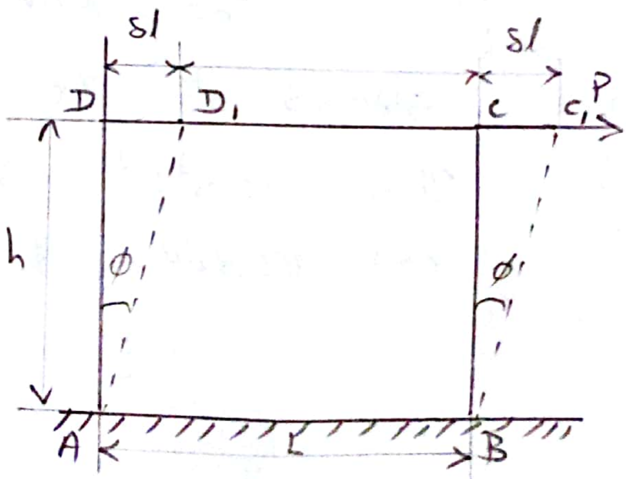
The ratio of decrease of length to the original length of the body is known as compressive strain.




Shear strain :-

The strain produced by shear stress is known as shear strain.

Consider a block as shown in fig. and fixed the bottom face. As the bottom face of the block is fixed, the face ABCD will be distorted to $ABC_1D_1$ through an angle $\phi$ as a result of force P.




$$\text{Shear strain}(\phi) = \frac{DD_1}{AD}$$

$$= \frac{dl}{h}.$$

## Volumetric Strain :-

The ratio of change in volume of the body to the original volume is known as volumetric strain.

$$e_v = \frac{\delta V}{V}$$

## HOOKE'S LAW

Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to the strain. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as modulus of elasticity (or) modulus of rigidity (or) Elastic moduli.

$$\sigma \propto e \Rightarrow \frac{\sigma}{e} = E$$

## MODULUS OF ELASTICITY (OR) YOUNG'S MODULUS

The ratio of tensile stress (or) compressive stress to the corresponding strain is a constant. This constant is known as youngs modulus (or) modulus of Elasticity and it is denoted by 'E'.

$$E = \frac{\text{Tensile (or) Compressive stress}}{\text{Tensile (or) Compressive strain}}$$

$$E = \frac{\sigma}{e}$$

MODULUS OF RIGIDITY (OR) SHEAR MODULUS :-

The ratio of shear stress to the corresponding shear strain within the elastic limit is known as modulus of Rigidity or, shear modulus. It is denoted by 'G'.

$$G = \frac{\text{Shear stress } (\tau)}{\text{shear strain } (e)}$$

BULK MODULUS :-

when a body is subjected to the mutually perpendicular like and equal direct stresses. The ratio of direct stress to the corresponding volumetric strain is known as Bulk modulus. It is denoted by 'K'

$$K = \frac{\text{Direct stress}}{\text{volumetric strain}} = \frac{\sigma}{\frac{\delta V}{V}}$$

FACTOR OF SAFETY :-

It is defined as the ratio of ultimate stress to the working (or permissible) stress. Is known as factor of safety. It is denoted by F.O.S.

$$F.O.S = \frac{\text{ultimate stress}}{\text{permissible stress}}$$

POISSON'S RATIO :-

The ratio of lateral strain to the longitudinal strain is known as poisson's ratio. It is denoted by '$\mu$'.

$$\text{poisson's ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\cancel{\delta d}}{\cancel{\ell d}} \quad \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}$$

$$\boxed{\mu = \frac{\delta d \times l}{d \times \delta l}}$$

= 9) An elastic rod 20 mm in diameter, 200 mm long extends by 0.25 mm under a tensile load of 40 KN. Find the intensity of stress, strain and the elastic modulus for the ma... of the rod.

Sol Given data.

Diameter of rod (d) = 20 mm

Length of rod ($l$) = 200 mm.

elongation ($\delta l$) = 0.25 mm

Tensile Load (P) = 40 KN

we know stress $(\sigma) = \frac{\text{Force (P)}}{\text{Area (A)}}$

Area of circular cross-section (A) $= \frac{\pi}{4}(20)^2$

$A = 314.15 \ mm^2$

$\sigma = \frac{40 \times 10^3}{314.15} = 127.32 \ N/mm^2$

strain(e) $= \frac{\delta l}{L} = \frac{0.25}{200} = 1.25 \times 10^{-3}$

$= 0.00125$

Elastic modulus (E) $= \frac{\sigma}{e}$

$= 101856 \ N/mm^2$

$= 1.01 \times 10^5 \ N/mm^2$

The safe stress, for a hollow steel Column which Carries an axial load of $2.1 \times 10^3$ KN is $125 \ MN/m^2$. If the external diameter of the Column is 30cm, determine the internal diameter.

Sol Given data :-

Axial Load (P) $= 2.1 \times 10^3$ KN

$= 2.1 \times 10^3 \times 10^3$ N

stress $(\sigma) = 125 \ MN/m^2$

$= 125 \times 10^6 \ N/m^2$

$$= \frac{125 \times \cancel{10^6}}{\cancel{10^6}} \, N/mm^2$$

$$\sigma = 125 \, N/mm^2$$

External diameter (D) = 30 cm =

$$= \frac{30}{100} = 0.3 m$$

$$= 30 \times 10 = 300 mm$$

we know

$$\sigma = \frac{P}{A}$$

Area of hollow cylinder (A) = $\frac{\pi}{4}(D^2 - d^2)$

$$125 = \frac{2.1 \times 10^6}{\frac{\pi}{4}(300^2 - d^2)} = \frac{2.1 \times 10^6 \times 4}{\pi(300^2 - d^2)}$$

$$125 \times \pi (300^2 - d^2) = 84 \times 10^5$$

$$35342917.35 - 125\pi d^2 = 84 \times 10^5$$

$$35342917.35 - 84 \times 10^5 = 125 \pi d^2$$

$$\frac{26942917.35}{125\pi} = d^2$$

$$d = \sqrt{68609.57}$$

$$\boxed{d = 261.93 mm}$$

Q) The ultimate stress, for a hollow steel column which carries an axial load of 1.9MN is 480 N/mm². If the external diameter of the column is 200 mm, determine the internal diameter. Take the factor of safety as 4.

Sol Given data:

Axial Load (P) = 1.9 MN
$= 1.9 \times 10^6$ N

Ultimate stress(σ) = 480 N/mm²

External diameter (D) = 200mm

Factor of safety (FOS) = 4.

Area of hollow Column = $\frac{\pi}{4}(D^2 - d^2)$

Factor of safety = $\frac{\text{Ultimate stress}}{\text{working stress}}$

working stress = $\frac{\text{Ultimate stress}}{\text{F.O.S}}$

$= \frac{480}{4} = 120$ N/mm²

$\sigma = \frac{P}{A} \Rightarrow 120 = \frac{1.9 \times 10^6}{\frac{\pi}{4}(200^2 - d^2)}$

$\pi \times 120(200^2 - d^2) = 1.9 \times 10^6 \times 4$

$15079644.74 - 120\pi d^2 = 76 \times 10^5$

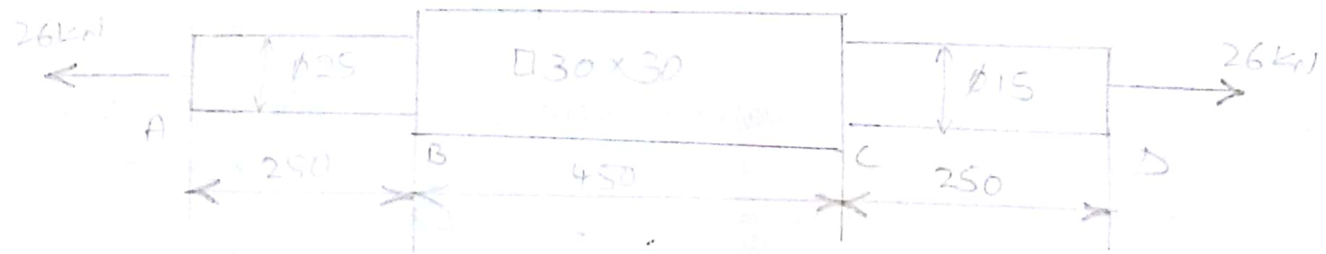
$$7479644.74 = 120\pi d^2$$

$$d^2 = \frac{7479644.74}{120 \times \pi}$$

$$d = \sqrt{19840.37}$$

$$\boxed{d = 140.85 \text{ mm}}$$

Q) A bar ABCD 950mm long is made up of three parts AB, BC and CD of lengths 250mm, 450 mm and 250 mm respectively. AB and CD are cylindrical having diameters 25 mm and 15 mm respectively. The rod BC is square section 30mm × 30 mm. The rod is subjected to a pull of 26 KN. Find i) The stresses in the three parts of the rod and (ii) The extension of the rod. Take $E = 2 \times 10^5$ N/mm².




Sol Given data :-

length of the bar ABCD = 950 mm

length of AB = 250 mm

BC = 450 mm

CD = 250 mm

Diameter of AB = 25mm

CD = 15mm

Area of the rod BC = 30×30.

pulling force (F) = 26 KN

young's modulus (E) = $2\times10^5$ N/mm²

$\sigma_{AB}$, $\sigma_{BC}$, $\sigma_{CD}$ & $\delta l_{AB}$, $\delta l_{BC}$, $\delta l_{CD}$ = ? (SI)

we know

$$\text{stress} = \frac{F}{A}$$

$$\sigma_{AB} = \frac{26\times10^3}{\frac{\pi}{4}(25)^2} = \frac{26000}{490.87}$$

$$\sigma_{AB} = 52.96 \ N/mm^2$$

$$\sigma_{BC} = \frac{26\times10^3}{30\times30} = \frac{26000}{900} = 28.88 \ N/mm^2$$

$$\sigma_{CD} = \frac{26\times10^3}{\frac{\pi}{4}(15)^2} = \frac{26000}{176.71} = 147.13 \ N/mm^2$$

we Also know

$$E = \frac{\sigma}{e} \Rightarrow \frac{\delta l}{l} = \frac{\sigma}{E}$$

$$\delta l_{AB} = \frac{52.96}{2\times10^5}\times 250 = \frac{13240}{2\times10^5}$$

$$\delta l_{AB} = 0.066 \ mm$$

$$\delta l_{BC} = \frac{\sigma_{BC}}{E_{BC}} \times L_{BC} \Rightarrow \frac{28.88}{2\times10^5} \times 450$$

$$= \frac{12996}{2\times105} = 0.06498\ mm$$

$$\delta l_{CD} = \frac{147.13}{2\times105} \times 250 = \frac{36782.5}{2\times10^5}$$

$$= 0.18391\ mm.$$

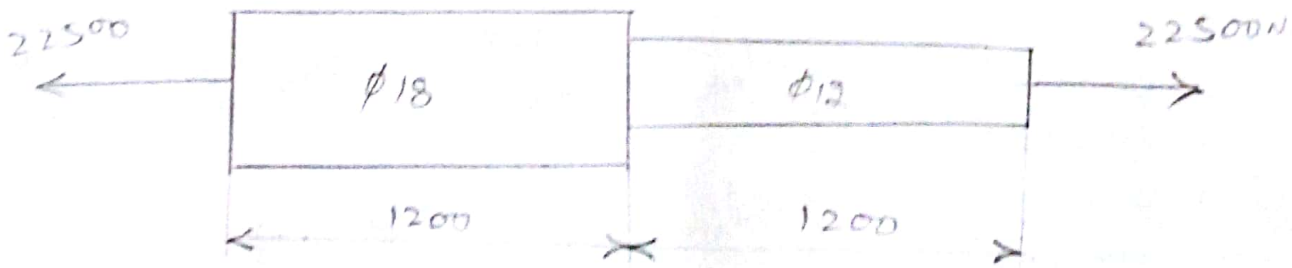
Total extension $= \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$

$$= 0.066 + 0.06498 + 0.18391$$

$$\boxed{\delta l = 0.3148\ mm}$$

3) A steel bar 2400 mm long is 18 mm in diameter for half the length and 12 mm diameter for the other half length. i) Find the extension of the bar when subjected to a Tensile force of 22500 N. (ii) For a steel bar of uniform diameter having the same length and volume what would be the extension when subjected to the same tensile force? Take $E = 2\times10^5\ N/mm^2$.

Sol Given data:



we know

$$\sigma = \frac{P}{A}$$

$$e = \frac{\delta l}{l}$$

$$E = \frac{\sigma}{e} = \frac{P/A}{\delta l / l}$$

$$E = \frac{Pl}{A\delta l}$$

$$\delta l = \frac{Pl}{AE}$$

$$\delta l = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2}$$

$$= \frac{22500 \times 1200}{\frac{\pi}{4}(18)^2 \times 2\times 10^5)} + \frac{22500 \times 1200}{\frac{\pi}{4}(12)^2 \times 2\times 10^5)}$$

$$= 135 \left[\frac{1}{254.46} + \frac{1}{113.09}\right]$$

$$= 135(0.0127)$$

$$\boxed{\delta l = 1.724\ mm}$$

ii) For uniform diameter having same length & volume.

$$L \times \frac{\pi}{4} d^2 = \frac{\pi}{4} d_1^2 \times L_1 + \frac{\pi}{4} d_2^2 \times L_2$$

$$2400 \times \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(18)^2 \times 1200 + \frac{\pi}{4}(12)^2 \times 1200$$

$$= 305362.80 + 135716.80$$

$$d^2 = \frac{441060 \times 4}{2400 \times \pi}$$

$$d = \sqrt{233.98}$$

$$\boxed{d = 15.29\ mm}$$

$$\delta l = \frac{Pl}{AE} = \frac{22500 \times 2400}{\frac{\pi}{4}(15.29)^2 \times 2\times 10^5} = \frac{54 \times 10^6}{36755000}$$

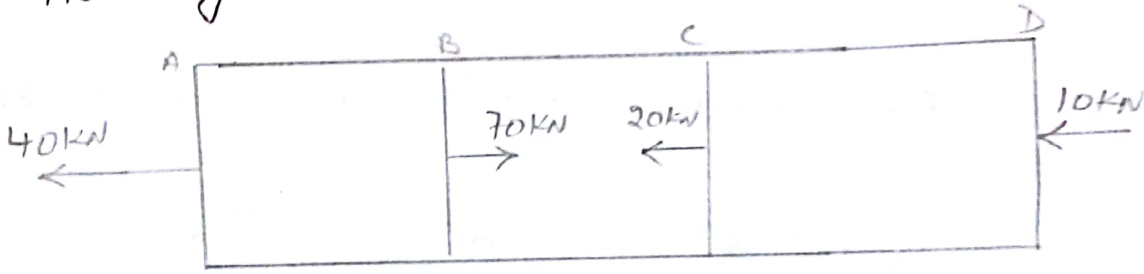
$$\boxed{\delta l = 1.46\ mm}$$

## Principle of superposition :-

When a number of Loads are acting on a body, the resulting strain will be the algebraic sum of strains caused by individual loads.

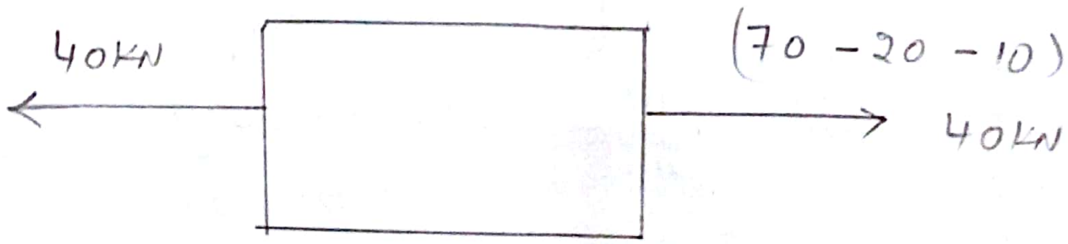
Then the total deformation of the body will be equal to the algebraic sum of deformations of the individual sections.

Q) A brass bar, having cross-section area of $900 mm^2$, is subjected to axial forces as shown in figure in which $AB = 0.6m$, $BC = 0.8m$ & $CD = 1m$. Find the elogation of the bar. Take $E = 1 \times 10^5 N/mm^2$.

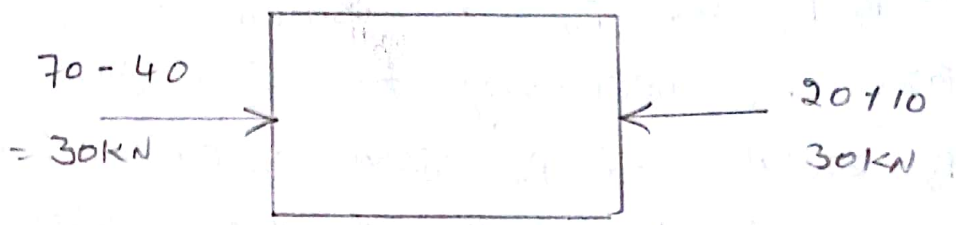



Sol Whenever the body is subjected to number of Loads at different sections along the length of the body, First we have to draw Free body diagram for individual sections.

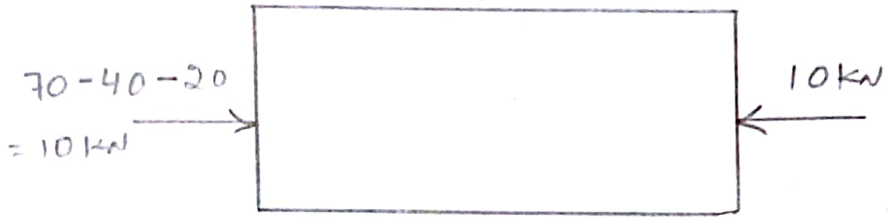
Let us Consider Block AB.



Block BC




Block CD




Now we have to calculate elongations in each block. For calculating elongation we know the formula i-e

$$\delta l_{AB} = \frac{PL}{AE} = \frac{40\times10^3 \times 0.6\times10^3}{900 \times 1\times10^5}$$

$= 0.26$ mm

$$\delta l_{BC} = \frac{PL}{AE} = \frac{30\times10^3 \times 0.8\times10^3}{900 \times 1\times10^5}$$

$= 0.26$ mm

$$\delta l_{CD} = \frac{PL}{AE} = \frac{10\times10^3 \times 1000}{900 \times 1\times10^5}$$

$= 0.111$ mm

Total elongation $= \delta l_1 + \delta l_2 + \delta l_3$

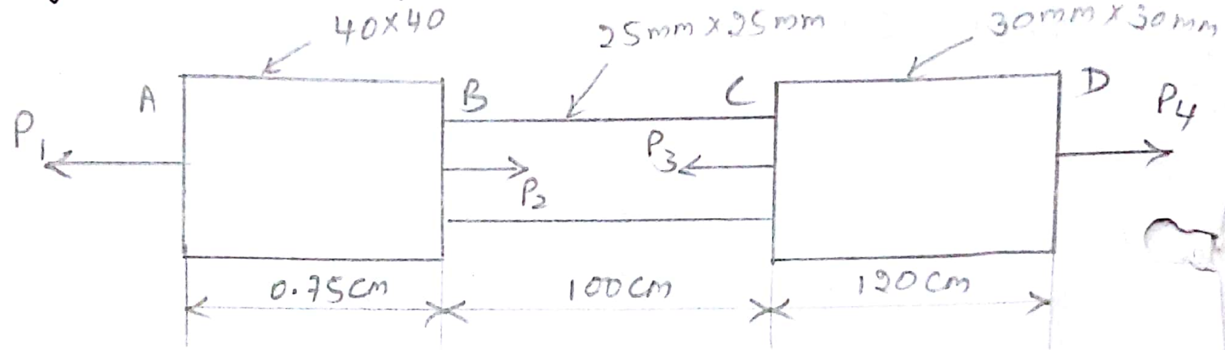
$= \delta l_{AB} + (-\delta l_{BC}) + (-\delta l_{CD})$

$= 0.26 - 0.26 - 0.1111$ mm

$\boxed{\delta l = -0.1111 \text{ mm}}$

-ve sign indicates there will be decrease in length of the

= 9) A member ABCD is subjected to point loads $P_1, P_2, P_3$ and $P_4$ as shown in fig calculate the force $P_3$ necessary for equilibrium if $P_1 = 120\,KN$, $P_2 = 220\,KN$, and $P_4 = 160\,KN$. Determine also the net change in the length of the member. Take $E = 200\,\frac{GN}{m^2}$.




Sol For calculating the $P_3$ value we have to Resolve the forces. Sum of Leftward forces = Sum of Rightward forces.

From the above fig we have

$$P_1 + P_3 = P_2 + P_4$$

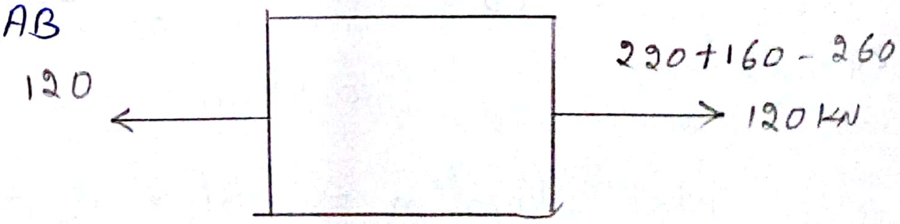
$$120 + P_3 = 220 + 160$$

$$P_3 = 380 - 120$$

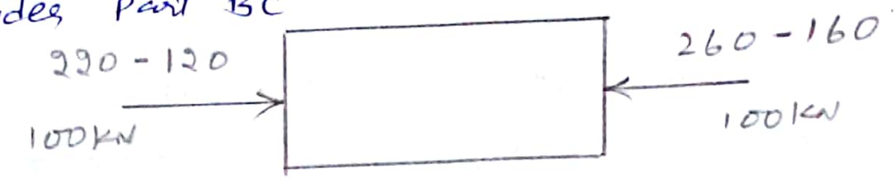
$$\boxed{P_3 = 260\,KN}$$

Now draw the Free body diagram for the above fig.

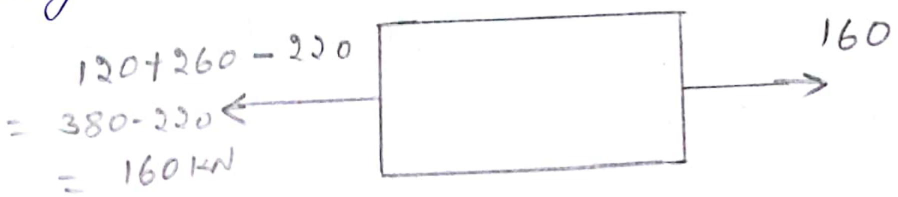
Consider part AB



Now Consider Part BC




Similarly for part CD




Now we have to Calculate change in the length for every part. we know.

$$\delta l = \frac{PL}{AE}$$

$$\delta l_{AB} = \frac{120\times10^3 \times 0.75\times100}{40\times40\times200\times10^3} = \frac{9\times10^5}{32\times10^7}$$

$$= 0.0028mm = 2.81\times10^{-3} mm$$

$$\delta l_{BC} = \frac{100\times10^3 \times 100\times10}{25\times25 \times 200\times10^3} = \frac{1\times10^8}{125\times10^6}$$

$$= 0.8mm$$

$$\delta l_{CD} = \frac{160\times10^3 \times 120\times10}{30\times30\times 200\times10^3} = \frac{192\times10^6}{18\times10^7}$$

$$= 1.066 mm$$

$$\delta l = \delta l_{AB} - \delta l_{BC} + \delta l_{CD}$$

$$= 2.81\times10^{-3} - 0.8 + 1.066$$

$$= 1.0688 - 0.8$$

$$\boxed{\delta l = 0.2688 mm}$$

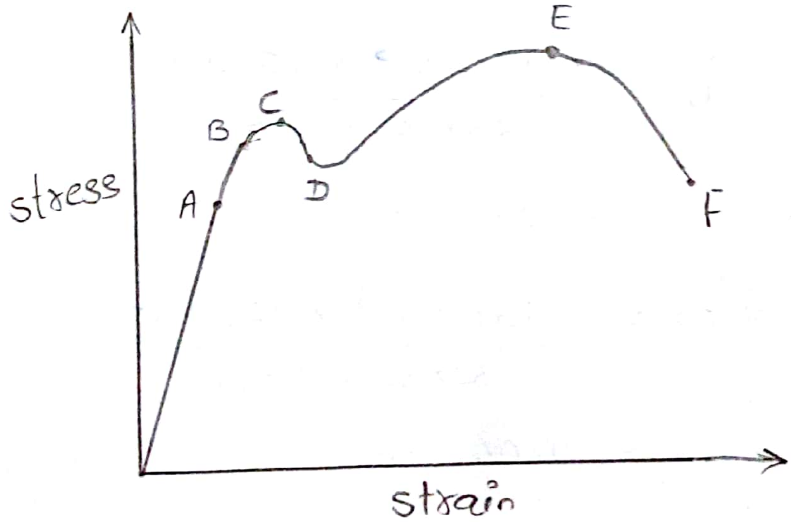
ELASTICITY :-

When an external Load acts on a body, the body tends to undergo some deformation. If the external force is removed and the body Come back to its original shape and size. This property is called Elasticity.

PLASTICITY :-

If the external force is so Large that the stress exceeds the elastic limit. If the force is removed the material will not return to its original shape and size is called plasticity.

STRESS - STRAIN DIAGRAM FOR MILD STEEL :-




The various points are observed on stress-strain curve.

(i) Limit of proportionality (A)

It is the point where stress is directly proportional to the strain upto the

MECHANICAL PROPERTIES :-

Strength :-

It is the ability of material to withstand / resist external Load without breaking (or) Fracture.

Ductility :-

It is the ability of material to deform under tensile load. It can drawn into thin wire without any rupture.

malleability :-

The ability of material to undergo plastic deformation under compressive load. malleable materials can be shaped by hammering or rolling. without any rupture.

Brittleness :-

The ability of material to fracture without appreciable deformation is called brittleness

Hardness :-

The ability of material to resist penetration (or) Indentation.

proportional limit

ii) ELASTIC LIMIT (B) :-

This point of is slightly beyond the limit of proportionality. If the material is stressed and then released, strain disappears completely and the original length is regained. This point is known as elastic limit.

iii) UPPER YIELD POINT (C) :-

At this point the load starts reducing and the extension increases. This phenomenon is called yielding of material.

iv) LOWER YIELD POINT (D) :-

At this point the stress remains the same but strain increases for some time.

v) ULTIMATE STRESS (E) :-

It is the point where the maximum stress the material can resist. At this stage cross sectional area at a particular section starts reducing very fast. This is called neck formation.

After this stage, load resisted and hence the stress developed starts reducing.

vi) BREAKING POINT (F) :-

It is the point where the stress at which finally the specimen fails is called breaking point.

Q) A tensile test was Conducted on a mild steel bar. The following data was obtained from the test.

i) Diameter of the steel bar = 3cm
ii) Gauge length of the bar = 20 cm
iii) Load at elastic Limit = 250 KN
iv) Extension at a Load of 150 KN = 0.21 mm
v) maximum Load = 380 KN
vi) Total Extension = 60mm.
vii) Diameter of the rod at the failure = 2.25 cm

Determine a) young's modulus b) The stress at elastic limit c) percentage elongation d) The percentage decrease in area.

Sol we know :- $\sigma = \frac{P}{A}$ ; $e = \frac{\delta l}{l}$ ; $E = \frac{\sigma}{e}$.

For Calculating the young's modulus we have to take the load within elastic limit. The value of elastic limit is not given in the question. So we can take the Load just beyond the elastic point. i.e 150 KN.

$$\sigma = \frac{150 \times 10^3}{\frac{\pi}{4}(30)^2} = \frac{150000}{706.85} = 212.20 \; N/mm^2$$

$$e = \frac{\delta l}{l} = \frac{0.21}{20 \times 10} = 1.05 \times 10^{-3}$$

$$E = \frac{\sigma}{e} = \frac{212.20}{1.05 \times 10^{-3}} = 202095.23 = 2.02 \times 10^5 \; N/mm^2$$

b) The Stress at Elastic Limit :-

$$\text{Stress} = \frac{\text{Load at elastic Limit}}{\text{Area}}$$

$$= \frac{250 \times 10^3}{706.85} = \cancel{290.01\ N/mm^2}$$

$$= 353.68\ N/mm^2.$$

c) percentage Elongation :-

$$\% = \frac{\text{Output}}{\text{Input}} = \frac{\text{Final length} - \text{Initial length}}{\text{Initial length}}$$

$$= \frac{60}{20 \times 10} = 0.3 \times 100 = 30\%$$

d) percentage decrease in area.

$$= \frac{\text{Initial (Original) Area} - \text{Final Area}}{\text{Initial Area}}$$

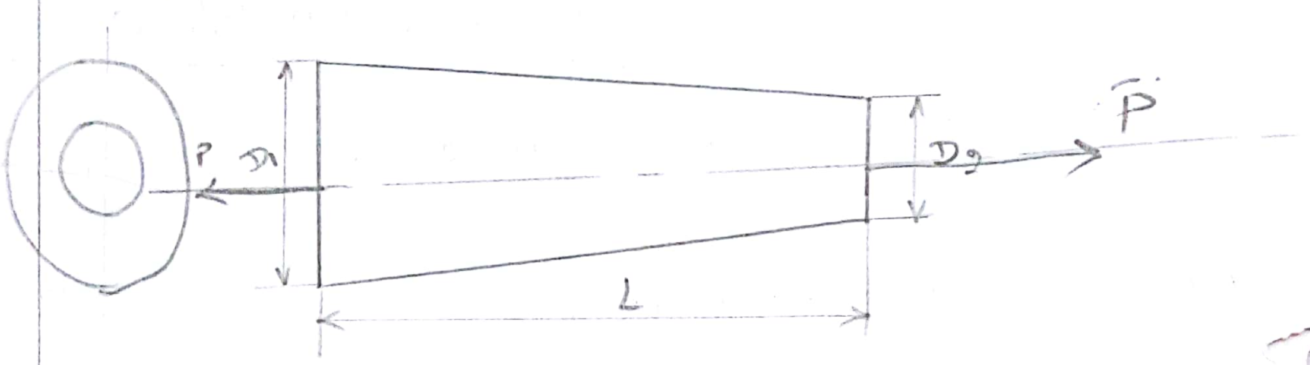
$$= \frac{\frac{\pi}{4}(30)^2 - \frac{\pi}{4}(22.5)^2}{\frac{\pi}{4}(30)^2} \times 100$$

$$= \frac{706.85 - 397.60}{706.85} \times 100 = \frac{\cancel{702.87}\ 309.25}{706.85} \times 100$$

$$= 0.4375 \times 100$$

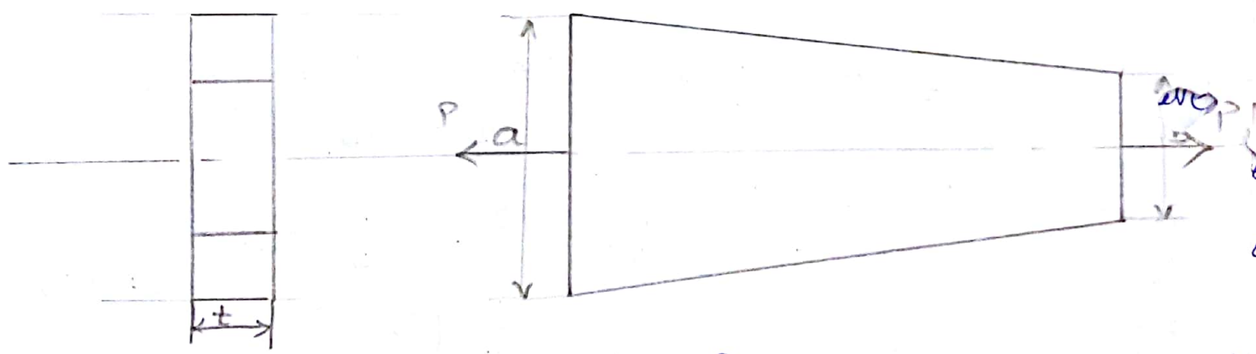
$$= 43.75\ \%$$

Total Elongation of a uniformly tapering circular rod of diameters $D_1$ & $D_2$ when the rod is subjected to an axial load.




$$\delta l = \frac{4PL}{\pi E D_1 D_2} = \frac{PL}{\frac{\pi (D_1 D_2)}{4} \times E}$$

Total elongation of a uniformly tapering rectangular bar when subjected to an axial load.




$$\delta l = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}$$

L = Total length of bar

a = width at bigger end

b = width at smaller end

t = thickness of the bar

E = young's modulus.

Q) Find the modulus of elasticity for a rod which tapers uniformly from 30mm to 15mm diameter in a length of 350mm. The rod is subjected to an axial load of 5.5KN and extension of the rod is 0.025mm.

Sol Given data.

Larger diameter $(D_1) = 30mm$

Smaller diameter $(D_2) = 15mm$.

Length $(L) = 350mm$.

Axial Load $(P) = 5.5KN$

Extension $(\delta l) = 0.025mm$

The above rod is in the taper form. Then we have

$$\delta l = \frac{4PL}{\pi E D_1 D_2}$$

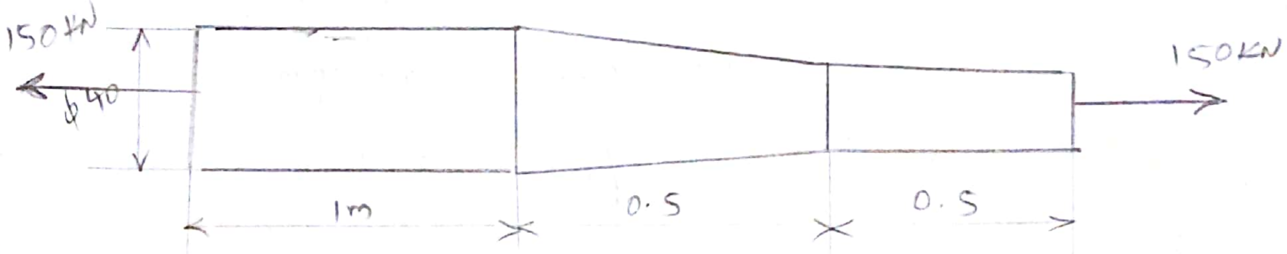
$$E = \frac{4 \times 5.5 \times 10^3 \times 350}{\pi \times 0.025 \times 30 \times 15}$$

$$= \frac{77 \times 10^5}{35.34}$$

$$= 2.17 \times 10^5 \ N/mm^2.$$

9) A 2m long steel bar has a uniform diameter of 40mm for a length of 1m from one end. For the next 0.5 m length the diameter decreases uniformly to d. For the remaining 0.5m length it has a uniform diameter of d mm. When a load of 150kN is applied, the observed extension is 2.4 mm. Determine the diameter d. Take modulus of elasticity for steel equal to 200 kN/mm²

Sol




$\delta l = 2.4$ mm

$d = ?$

$E = 200\ kN/mm^2 = 2 \times 10^5\ N/mm^2$

we know

$$\delta l_1 = \frac{P.L}{AE} = \frac{150 \times 10^3 \times 1000}{\frac{\pi}{4}(40)^2 \times 2 \times 10^5} = 0.5968$$

$$\delta l_2 = \frac{150 \times 10^3 \times 0.5 \times 10^3 \times 4}{\pi\, 40 \times d \times 2 \times 10^5} = \frac{11.93662}{d}\ mm$$

$$\delta l_3 = \frac{150 \times 10^3 \times 0.5 \times 10^3 \times 4}{\pi d^2 \times 2 \times 10^5} = \frac{477.46}{d^2}\ mm$$

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Total elongation $(\delta l) = \delta l_1 + \delta l_2 + \delta l_3$

$$= 0.5968 + \frac{11.93662}{d} + \frac{477.46}{d^2}$$

$$2.4 = 0.5968 + \frac{11.93662d^2 + 477.46d}{d^3}$$

$$d^3(2.4 - 0.5968) = 11.9366d^2 + 477.46d$$

$$1.8032d^3 - 11.9366d^2 - 477.46d$$

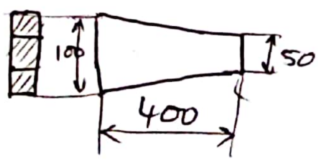
$$\boxed{d = 19.91\,mm}$$

9) The extension in a rectangular steel bar of length 400mm and thickness 10mm, is found to be 0.21mm. The bar tapers uniformly in width from 100mm to 50mm. If E for the bar is $2 \times 10^5$ N/mm². Determine the axial load on the bar.

Sol Given data:- Length of the bar(L) = 400mm

Thickness (t) = 10mm

Extension $(\delta l)$ = 0.21mm




width at bigger end (a) = 100mm

width at smaller end (b) = 50mm

young's modulus (E) = $2 \times 10^5$ N/mm²

we know

$$\delta l = \frac{PL}{Et(a-b)} \log_e\left(\frac{a}{b}\right)$$

$$P = \frac{0.21 \times 10 \times (100-50) \times 2\times10^5}{400 \log_e \left(\frac{100}{50}\right)}$$

$$= \frac{21\times10^6}{277.25}$$

$$\boxed{P = 75741.48\,N}$$

9) A straight bar of steel rectangular in section is 3m long and ~~15mm thick~~. The width of the rod varies from 100mm at one end to 40mm at the other. If the rod is subjected to an axial tensile load of 40KN, and the extension of the bar is limited to 0.5mm. find the min thickness of the bar. Take $E = 2\times10^5\ N/mm^2$.

sol Given data

length (L) = 3m = 3000mm

Thickness (t) = 15mm

width (a) = 100mm

width at smaller end (b) = 40mm

Tensile load (P) = 40KN = 40000N

Extension ($\delta l$) = 0.5mm

young's modulus (E) = $2\times10^5$

we know

$$\delta l = \frac{PL}{E\,t\,(a-b)} \log_e\left(\frac{a}{b}\right)$$

$$t = \frac{40\times10^3 \times 3000}{2\times10^5 \times 0.5 \times (100-40)} \log_e\left(\frac{100}{40}\right)$$

$$= \frac{12\times10^7}{6\times10^6} \log_e(2.5)$$

$$= 20 \times 0.9162$$

$$\boxed{t = 18.32\,mm}$$

4) ~~COMPOSITE~~ BARS OF COMPOSITE SECTIONS :-

If a member is made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile (or) compressive loads is called composite bar.

For composite bar the following two points are important.

1) The extension or compression in each bar is equal, hence strain in each bar is equal.

2) The total external load on composite bar is equal to the sum of loads carried by different material.

Then

$$P = P_1 + P_2$$

As well as strain will be same for each bar

$$e_1 = e_2$$

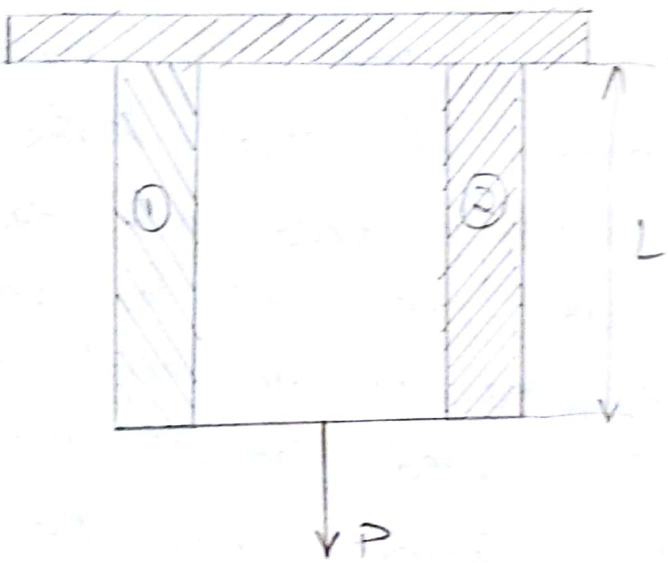
$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E}$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$$

$\therefore \frac{E_1}{E_2}$ - is called the modular ratio.




Q) A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4cm. The composite bar is then subjected to an axial pull of 45kN. If the length of each bar is equal to 15cm, Determine

1) The stresses in the rod and tube

2) Load carried by each bar.

Take $E_s = 2.1 \times 10^5\ N/mm^2$ and $E_c = 1.1 \times 10^5\ N/mm^2$.

Sol Given data :.

Dia of steel rod ($d_s$) = 3cm

External dia of Cu tube ($A_o$) = 5cm

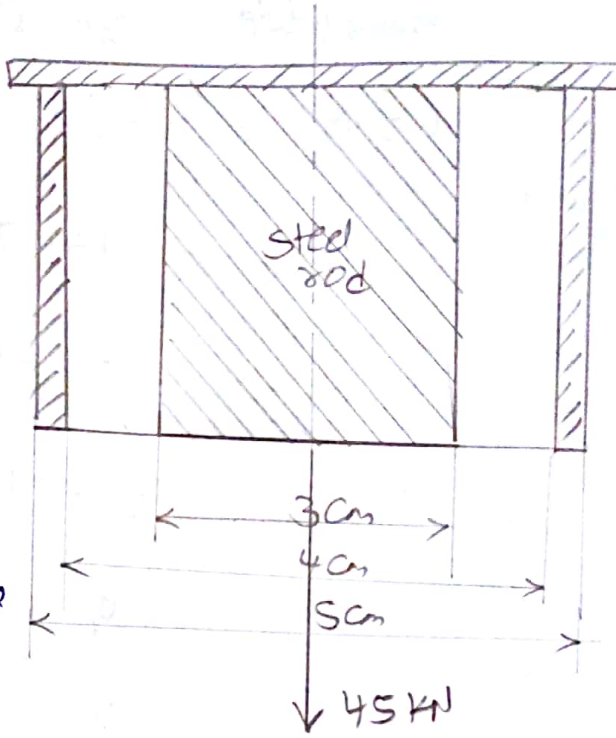
Internal dia of cu tube ($A_i$) = 4cm

Axial Load (P) = 45KN

length of each bar (L) = 15cm

$E_S = 2.1 \times 10^5 \, N/mm^2$

$E_C = 1.1 \times 10^5 \, N/mm^2$




As we know

$$P = P_1 + P_2$$

$$\sigma = \frac{P}{A}$$

$$\boxed{P = \sigma \cdot A}$$

But

$$P = \sigma_S A_S + \sigma_C A_C$$

Also

$$45000 = \sigma_S \times \frac{\pi}{4}(30)^2 + \sigma_C \frac{\pi}{4}(50^2 - 40^2)$$

$$45000 = 706.85\,\sigma_S + \cancel{1256.63}\ 706.85\,\sigma_C \quad — (1)$$

we also know

$$e_1 = e_2$$

$$\boxed{\frac{\sigma}{e} = E}$$

$$\frac{\sigma_S}{E_S} = \frac{\sigma_C}{E_C}$$

$$\sigma_S = \sigma_C \times \frac{2.1 \times 10^5}{1.1 \times 10^5}$$

$$\boxed{\sigma_S = 1.9090\,\sigma_C} \quad — (2)$$

substitute eqn ② value in eqn ①

$$45000 = 706.85 \times 1.9090\,\sigma_c + \cancel{1256.63}^{706.85}\,\sigma_c$$

$$= 1349.44\,\sigma_c + \cancel{1256.63}^{706.85}\,\sigma_c$$

$$= 2056.29\,\sigma_c$$

$\cancel{\sigma_c = 17.26\ N/mm^2}$ $\Rightarrow \sigma_c = \frac{45000}{2056.29}$

$$\boxed{\sigma_c = 21.88\ N/mm^2}$$

$$\sigma_s = 1.9090 \times \sigma_c \Rightarrow 1.9090 \times 21.88$$

$$\boxed{\sigma_s = 41.77\ N/mm^2}$$

Load Carried by each bar:-

Load Carried by steel bar $(P_s) = \sigma_s \times A_s$

$$= 41.77 \times \frac{\pi}{4}(30)^2$$

$$\boxed{P_s = 29.52\,kN}$$

Load Carried by Copper tube $(P_c) = \sigma_c \times A_c$

$$= 21.88 \times \frac{\pi}{4}(50^2 - 30^2)$$

$$\boxed{P_c = 15.46\,kN}$$

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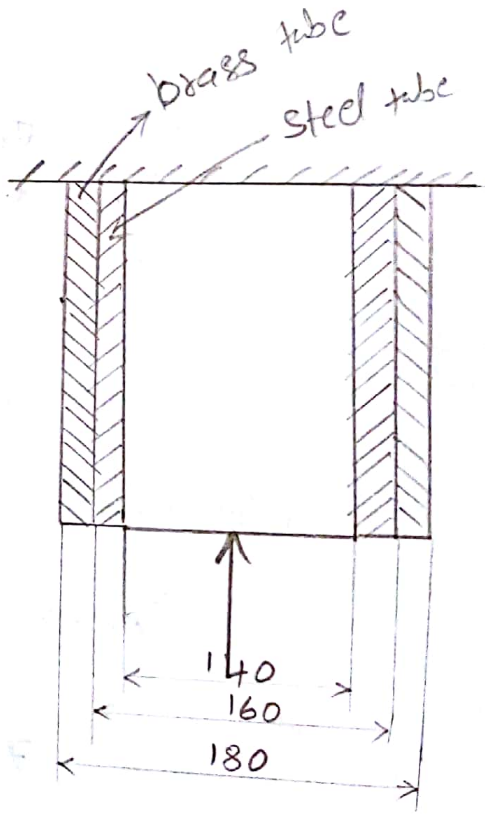
9) A Compound tube Consists of a steel tube 140mm internal diameter and 160mm external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tubes carries an axial load of 900KN. Find the stresses and load carried by each tube and the amount it shortens. length of each tube is 140mm. Take E for steel as $2\times10^5\ N/mm^2$ and for brass as $1\times10^5\ N/mm^2$.

Sol Given data




Steel tube Internal diameter $(D_i) = 140mm$

steel tube External diameter $(D_o) = 160mm$.

Brass tube Internal diameter $(D_i) = 160mm$

Brass tube External diameter $(D_o) = 180mm$

length of steel bar = length of Brass bar

Axial Load $(P) = 900KN$

Length $(L) = 140mm$

young's modulus for steel $(E_s) = 2\times10^5 N/mm^2$

young's modulus for Brass $(E_b) = 1\times10^5 N/mm^2$

we know

$$P = P_s + P_b$$

$$\sigma = \frac{P}{A}$$

$$P = \sigma . A$$

$$P = \sigma_s A_s + \sigma_b A_b$$

$$900 \times 10^3 = \sigma_s \frac{\pi}{4}(160^2 - 140^2) + \sigma_b \frac{\pi}{4}(180^2 - 160^2)$$

$$9 \times 10^5 = 4712.38\,\sigma_s + 5340.70\,\sigma_b \quad \text{— } (1)$$

we Also know for Composite bar

strain in brass tube = strain in steel tube

$$e_b = e_s$$

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E}$$

$$\frac{\sigma_b}{E_b} = \frac{\sigma_s}{E_s}$$

$$\sigma_b = \frac{E_b}{E_s} \times \sigma_s \Rightarrow \frac{1 \times \cancel{10^5}}{2 \times \cancel{10^5}} \times \sigma_s$$

$$\boxed{\sigma_b = 0.5\,\sigma_s} \text{ ——— } (2)$$

substitute eqn (2) in eqn (1)

$$9 \times 10^5 = 4712.38\,\sigma_s + 5340.7 \times 0.5\,\sigma_s$$

$$= 4712.38\,\sigma_s + 2670.35\,\sigma_s$$

$$9 \times 10^5 = 7382.73\,\sigma_s$$

$$\sigma_s = \frac{9 \times 10^5}{7382.73} = 121.9\ N/mm^2$$

$$\boxed{\sigma_s = 121.9\ N/mm^2}$$

$$\sigma_b = 0.5\,\sigma_s = 0.5 \times 121.9$$

$$\boxed{\sigma_b = 60.95\ N/mm^2}$$

Now calculate the load carried by each tube.

Load carried by Brass tube ($P_b$) = $\sigma_b \times A_b$

$$= 60.95 \times \frac{\pi}{4}(180^2 - 160^2)$$

$$= 60.95 \times 5340.70$$

$$\boxed{P_b = 325.51\ KN}$$

Similarly load carried by steel tube.

$$P_s = \sigma_s \times A_s$$

$$= 121.9 \times \frac{\pi}{4}(160^2 - 140^2)$$

$$= 121.9 \times 4712.38$$

$$\boxed{P_s = 574.44\ KN}$$

Now we have to calculate the amount of it shortens

we know strain in Brass tube = strain in steel tube

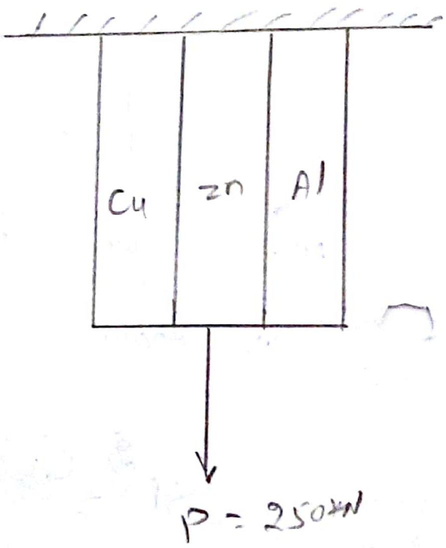
$$\delta l = \frac{P_b L_b}{A_b E_b} = \frac{325.51 \times 10^3 \times 140}{5340.70 \times 1 \times 10^5}$$

$$= \frac{45571400}{53407 \times 10^4}$$

$$\boxed{\delta l = 0.085\ mm}$$

9) Three bars made of Copper, zinc and aluminium are equal of length and have cross-section 500, 750 and 1000 square mm respectively. They are rigidly connected at their ends. If this compound member is subjected to a longitudinal pull of 250 KN, estimate the proportional of the load carried on each rod and the induced stresses. Take the value of E for Copper = $1.3 \times 10^5$ N/mm² for zinc = $1.0 \times 10^5$ N/mm² and for aluminium = $0.8 \times 10^5$ N/mm².

Sol Given data




Area of Copper bar = 500 mm²

Area of zinc bar = 750 mm²

Area of Aluminium bar = 1000 mm²

pulling force (P) = 250 KN

youngs modulus for Cu = $1.3 \times 10^5$ N/mm²

$E_z = 1 \times 10^5$ N/mm²

$E_{Al} = 0.8 \times 10^5$ N/mm².

we know $P = P_c + P_z + P_{Al}$

$$= \sigma_c \times A_c + \sigma_z \times A_z + \sigma_{Al} \times A_{Al}$$

$$250 \times 10^3 = \sigma_c \times 500 + \sigma_z \, 750 + \sigma_{Al} \times 1000 \quad \text{—— (1)}$$

we also know

strain in Copper = strain in zinc = strain in aluminium

$$\frac{\sigma_c}{E_c} = \frac{\sigma_{zn}}{E_{zn}} = \frac{\sigma_{Al}}{E_{Al}}$$

$\boxed{\therefore \frac{\sigma}{e} = E}$

$\boxed{e = \frac{\sigma}{E}}$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_{zn}}{E_{zn}}$$

$$\sigma_c = \frac{\sigma_{zn}}{E_{zn}} \times E_c \Rightarrow \frac{E_c}{E_{zn}} \times \sigma_{zn} \Rightarrow \frac{1.3 \times 10^5}{1 \times 10^5} \sigma_{zn}$$

$$\boxed{\sigma_c = 1.3\, \sigma_{zn}} \quad \text{--- } (2)$$

$$\frac{\sigma_{Al}}{E_{Al}} = \frac{\sigma_{zn}}{E_{zn}}$$

$$\sigma_{Al} = \frac{E_{Al}}{E_{zn}} \times \sigma_{zn} \Rightarrow \frac{0.8 \times 10^5}{1 \times 10^5} \times \sigma_{zn}$$

$$\boxed{\sigma_{Al} = 0.8\, \sigma_{zn}} \quad \text{--- } (3)$$

substitute equation (2) & (3) in equation (1) to get $\sigma_{zn}$ value.

$$250000 = (1.3\, \sigma_{zn})\, 500 + 750\, \sigma_z + (0.8\, \sigma_{zn})(1000)$$

$$= 650\, \sigma_{zn} + 750\, \sigma_{zn} + 800\, \sigma_{zn}$$

$$250000 = 2200\, \sigma_{zn}$$

$$\boxed{\sigma_{zn} = 113.63\ N/mm^2}$$

substitute $\sigma_{zn}$ value in equation. (3) to get $\sigma_{Al}$

$$\sigma_{Al} = 0.8\,\sigma_{zn}$$

$$= (0.8)(113.63)$$

$$\boxed{\sigma_{Al} = 90.90\ N/mm^2}$$

$$\sigma_c = 1.3\,\sigma_{zn}$$

$$= 1.3 \times 113.63$$

$$\boxed{\sigma_c = 147.71\ N/mm^2.}$$

Load Carried by Copper $(P_c) = \sigma_c \times A_c$

$$P_c = 147.71 \times 500 = 73855\ N$$

$$\boxed{P_c = 73.85\ KN}$$

Load Carried by zinc rod $(P_{zn}) = \sigma_{zn} \times A_{zn}$

$$= 113.63 \times 750 \Rightarrow 85222.5\ N$$

$$\boxed{P_{zn} = 85.22\ KN}$$

load Carried by Aluminium Rod $(P_{Al}) = \sigma_{Al} \times A_{Al}$

$$= 90.9 \times 1000$$

$$= 90900\ N$$

$$\boxed{P_{Al} = 90.9\ KN}$$

THERMAL STRESSES (OR) TEMPERATURE STRESSES :-

When the temperature of a material changes there will be corresponding change in its dimensions. When a member is free to expand or contract due to the rise or fall of temperature. no stress will be induced in the member.

— But if the natural change in length due to rise or fall of temperature be prevented, stress will be offered.

Consider a body which is heated to a certain temperature.

let $L$ = original length of the body

$T$ = Rise in temperature

$E$ = young's modulus

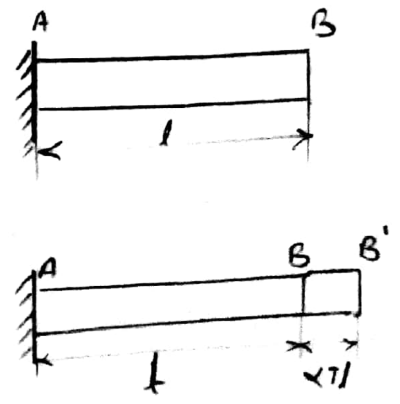
$\alpha$ = Co-efficient of linear expansion.

$dl$ = Extension of the rod due to rise of temperature.

If the rod is free to expand, then the extension of the rod is given by

$$dl = \alpha T L.$$

Suppose a bar is fixed at one end, then the rod tends to expands freely by $\alpha T l$ i.e $BB' = \alpha T l$

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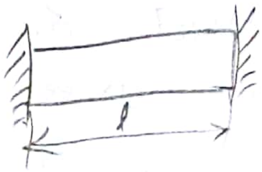
Let now an external load is applied at B so that the rod is decreased in length from $(l+\alpha T l)$ to $l$.
stress is developed at B

Then Compressive strain $= \dfrac{\alpha \cdot T l}{l} = \alpha T$

$\dfrac{\text{stress}}{\text{strain}} = E$

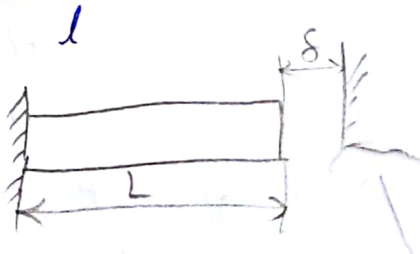


stress $= \alpha T E$

STRESS AND STRAIN WHEN THE SUPPORTS YIELD:-

suppose a rod of length $(l)$, when subjected to rise of temperature is permitted to expand only by $\delta$, then the temperature strain

$e = \dfrac{\text{Expansion prevented}}{\text{original length}} = \dfrac{\alpha T l - \delta}{l}$



stress $= e \times E = \dfrac{(\alpha T l - \delta)}{l} \times E$

Q) A rod is 2m long at 10°c. Find the expansion of the rod when the temperature is raised to 80°c. If this expansion is prevented find the stress in the material. Take $E = 1 \times 10^5 \, N/mm^2$ and $\alpha = 0.000012$ (or) $1.2 \times 10^{-5}$ per °c.

Sol Given data

length of the rod $(l) = 2m$

Initial temperature$(T_1) = 10°c$

Final temperature $(T_2) = 80°c$

Young's modulus $(E) = 1\times10^5\ N/mm^2$

Co-efficient of linear expansion$(\alpha) = 1.2\times10^{-5}\ °c$

we know.

Free expansion $= \alpha T l$

$= 1.2\times10^{-5}(80-10)\times 2000$

$\boxed{\delta l = 1.68\,mm}$

stress in the material $(\sigma) = e E$ $\quad \because e = \alpha T$

$= \alpha T E$

$= 1.2\times10^{-5}(80-10)\times 1\times10^5$

$\boxed{\sigma = 84\ N/mm^2}$

7) A steel rod 20m long at a temperature of 20°c. Find the free expansion of the rod when the temperature is raised by 65°c. find the temperature stress produced (i) when the expansion of the rod is prevented (ii) when the rod is permitted to expand by 5.8mm. Take $\alpha = 12\times10^{-6}$ per °c and $E = 2\times10^5\ N/mm^2$.

sol Given data:

length of rod $(l) = 20\,m = 20000\,mm$

Temperature rise $(T) = 65°C$

Expand $(\delta) = 5.8$ mm.

Coefficient of linear expansion $(\alpha) = 12 \times 10^{-6}$ per °C

Young's modulus $(E) = 1 \times 10^{5}$ N/mm²

i) Free expansion $(\delta l) = \alpha T l$

$= 12 \times 10^{-6} \times (65 \overset{20}{-}) \ 2 \times 10^{4}$

$= 12 \times 10^{-6} \times (45) \ 2 \times 10^{4}$

$= 10.8$ mm.

i) when the expansion of the rod prevented

$\sigma = e \times E \Rightarrow \alpha T E$

$= 12 \times 10^{-6} \times (45) \times 2 \times 10^{5}$

$\boxed{\sigma = 108 \ N/mm^2}$

ii) when the rod is permitted to expand by 5.8 mm.

$$e = \frac{\alpha T l - \delta}{l} = \frac{10.8 - 5.8}{20 \times 10^{3}} = 2.5 \times 10^{-4}$$

$\sigma = e \times E = 2.5 \times 10^{-4} \times 2 \times 10^{5}$

$\boxed{\sigma = 50 \ N/mm^2.}$

9) A steel rod of 3cm diameter and 5m long is connected to two grips and the rod is maintained at a temperature of 95°c. Determine the stress and pull exerted when the temperature falls to 30°c if.

i) The ends do not yield and
ii) The ends yield by 0.12 cm.

Take $E = 2\times10^5 \ MN/m^2$ & $\alpha = 12\times10^{-6}$ per °c

Sol Given data :-

Diameter of steel rod (d) = 3cm
= 3×10 mm

Length of steel rod (L) = 5m
= 5000 mm.

Higher temp ($T_2$) = 95°c

Lower temp ($T_1$) = 30°c

Yield ($\delta$) = 0.12cm ⇒ ~~0.012mm~~
= 1.2 mm

Young's modulus (E) = $2\times10^5 \ N/mm^2$

Coefficient of linear expansion ($\alpha$) = $12\times10^{-6}$ per °c

i) when the ends do not yield

Stress = $\alpha T E$

$= 12\times10^{-6}\times(95-30)\times2\times10^5$

$= 156 \ N/mm^2$.

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pull in the rod $= \sigma \times A$

$= 156 \times \frac{\pi}{4}(30)^2$

$= 156 \times 706.85$

$\boxed{P = 110.26 \, KN}$

when the ends yield by 0.12 cm (or) 1.2 mm

we have strain $= \frac{\alpha T L - \delta}{l}$

$= \frac{12 \times 10^{-6} \times (95-30) \times 5000 - 1.2}{5000}$

$= \frac{3.9 - 1.2}{5000} = \frac{2.7}{5 \times 10^3}$

$= 5.4 \times 10^{-4}$

stress $= e \times E \Rightarrow 5.4 \times 10^{-4} \times 2 \times 10^5$

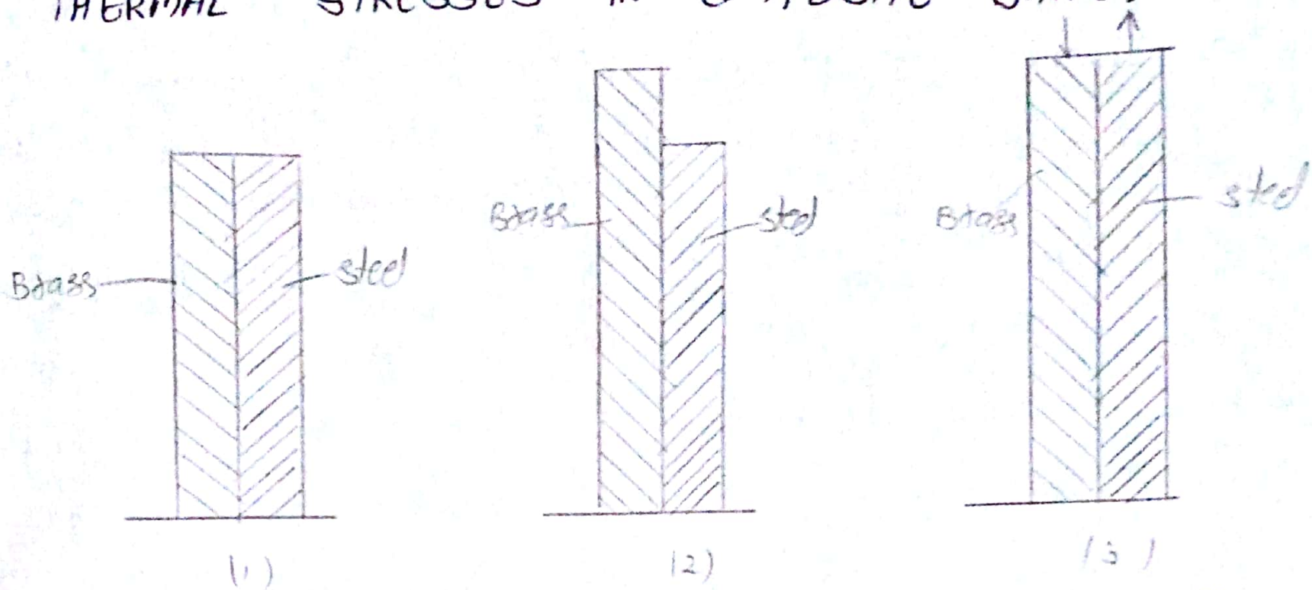
$\boxed{\sigma = 108 \, N/mm^2}$

$P = \sigma \times A$

$= 108 \times 706.85$

$\boxed{P = 76.33 \, KN}$

THERMAL STRESSES IN COMPOSITE BARS:-




- The above diagram (1) shows a Composite bar consisting of two members, a bar of brass and another of steel.
- Let the Composite bar be heated through some temperature. If the members are free to expand then there will be no stresses will be induced in the members.
- But the two members are rigidly fixed and hence the Composite bar as a whole will expand by same amount.
- As the co-efficient of linear expansion of brass is more than that of the steel. the brass member will expand more than ~~that~~ the steel as shown in fig (2)
- But if both the members are not free to expand then the stress induced in the brass will be Compressive whereas the stress

in steel will be tensile as shown in fig (3)

- Hence the Load acting on the brass will be Compressive whereas on the steel the load will be tensile.
- For the equilibrium of the system Compression in Copper should be equal to tension in the steel.

1) Load on the brass bar = Load on the steel.

$$P_b = P_s$$

$$\boxed{P = \sigma . A}$$

$$\sigma_b . A_b = \sigma_s . A_s$$

2) we Also know

Expansion of steel = Expansion of Brass

But Actual expansion of steel

= Free expansion of steel + Expansion due to Tensile stress

$$= \alpha_s T L + \frac{\sigma_s}{E_s} \times L$$

And Actual Expansion of Copper

= Free expansion of Cu ~~+ Expansion~~ − Contraction due to Compressive stress induced in brass

$$\alpha_s T L + \frac{\sigma_s}{E_s} \times L = \alpha_b T L - \frac{\sigma_b}{E_b} \times L$$

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_b T - \frac{\sigma_b}{E_b}$$

9) A steel rod of 20mm diameter passes centrally through a copper tube of 50mm external diameter and 40mm internal diameter. The tube is closed at each each end by rigid plates. The nuts are tightened lightly on the projecting parts of the rod. If the temperature of the assembly is raised by 50°c. Calculate the stresses developed in the copper and steel. Take E for steel and copper as 200 $GN/m^2$ and 100 $GN/m^2$ and α for steel & copper as $12 \times 10^{-6}$ per °c and $18 \times 10^{-6}$ per °c.

Sol Given data :-

Diameter of steel rod (d) = 20mm

External dia of Cu tube ($D_o$) = 50mm

Internal dia of Cu tube ($D_i$) = 40mm

Rise in temperature (T) = 50°c

$\sigma_s, \sigma_c$ = ?

Young's modulus of steel ($E_s$) = 200 $GN/m^2$

Youngs modulus of copper ($E_c$) = 100 $GN/m^2$

Co-efficient of linear expansion for steel = $12 \times 10^{-6}$ per °c

" " " " for copper = $18 \times 10^{-6}$ per °c

we know

Tensile load on steel ($P_s$) = ($P_c$) (Compressive load on Copper)

$$\sigma_s \cdot A_s = \sigma_c \cdot A_c$$

$$\sigma_s \times \frac{\pi}{4}(20)^2 = \sigma_c \times \frac{\pi}{4}(50^2 - 40^2)$$

$$\sigma_s \cdot 314.15 = 706.85\,\sigma_c$$

$$\boxed{\sigma_s = 2.24\,\sigma_c} \quad \text{—— (1)}$$

we Also know.

Actual expansion of steel = Actual expansion of Copper

$$\alpha_s T \cancel{L} + \frac{\sigma_s}{E_s} \times \cancel{L} = \alpha_c T \cancel{L} - \frac{\sigma_c}{E_c} \times \cancel{L}$$

$$12 \times 10^{-6} \times 50 + \frac{2.24\,\sigma_c}{2 \times 10^5} = 18 \times 10^{-6} \times 50 - \frac{\sigma_c}{1 \times 10^5}$$

$$6 \times 10^{-4} + 1.12 \times 10^{-5}\,\sigma_c = 9 \times 10^{-4} - 1 \times 10^{-5}\,\sigma_c$$

$$\sigma_c\left[1.12 \times 10^{-5} + 1 \times 10^{-5}\right] = 9 \times 10^{-4} - 6 \times 10^{-4}$$

$$\sigma_c = \frac{3 \times 10^{-4}}{2.12 \times 10^{-5}}$$

$$\boxed{\sigma_c = 14.15\ N/mm^2}$$

$$\sigma_s = 2.24\,\sigma_c \Rightarrow 2.24 \times 14.15$$

$$\boxed{\sigma_s = 31.69\ N/mm^2}$$

3) A steel tube of 30 mm external diameter and 25 mm internal diameter encloses a gun metal rod of 20 mm diameter to which it is rigidly joined at each end the temperature of the whole assembly is raised to 140°c and the nuts on the rod are then screwed lightly on the ends of the tube. Find the intensity of stress in the rod when the common temperature has fallen to 30°c. Take

$E_s = 2.1 \times 10^5 \ N/mm^2$ ; $E_g = 1 \times 10^5 \ N/mm^2$.

$\alpha_s = 12 \times 10^{-6}$ per °c ; $\alpha_g = 20 \times 10^{-6}$ per °c

Sol Given data :-

External dia of steel tube $(D_o)$ = 30mm

Internal dia of steel tube $(D_i)$ = 25mm.

Dia of gun metal rod $(D)$ = 20mm

Temperature raised $(T_f)$ = 140°c

Temperature fall $(T_i)$ = 30°c

youngs modulus for steel $(E_s) = 2.1 \times 10^5$

$E_g = 1 \times 10^5 \ N/mm^2$

$\alpha_s = 12 \times 10^{-6}$ per °c

$\alpha_g = 20 \times 10^{-6}$ per °c

we know

$$P_s = P_g$$

$$\sigma_s \cdot A_s = \sigma_g \cdot A_g$$

$$\sigma_s \times \frac{\pi}{4}(30^2 - 25^2) = \sigma_g \cdot \frac{\pi}{4}(20)^2$$

$$= \frac{314.15}{215.98}$$

$$\boxed{\sigma_s = 1.45\ \sigma_g} \text{ ——— } (1)$$

$$e_s = e_g$$

$$\delta l_s = \delta l_g$$

$$\alpha_s T_s L + \frac{\sigma_s}{E_s} \times L = \alpha_g T_g L - \frac{\sigma_g}{E_g} \times L$$

$$12 \times 10^{-6} \times (140-30) + \frac{1.45\sigma_g}{2.1 \times 10^5} = 20 \times 10^{-6} \times (140-30) - \frac{\sigma_g}{1 \times 10^5}$$

$$1.32 \times 10^{-3} + 6.90 \times 10^{-6}\ \sigma_g = 2.2 \times 10^{-3} - 1 \times 10^{-5}\ \sigma_g$$

$$\sigma_g \left[6.90 \times 10^{-6} + 1 \times 10^{-5}\right] = \frac{2.2 \times 10^{-3} - 1.32 \times 10^{-3}}{\cancel{1.32 \times 10^{-3}}}$$

$$= \frac{\cancel{1.66}\ 8.8 \times 10^{-4}}{1.69 \times 10^{-5}}$$

$$\boxed{\sigma_g = 52.07\ N/mm^2}$$

$$\sigma_s = 1.45\ \sigma_g$$

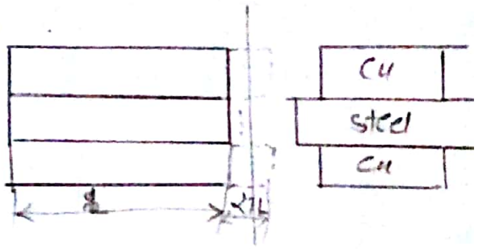
$$= 1.45 \times 52.07$$

$$\boxed{\sigma_s = 75.6\ N/mm^2}$$

Q) A Compound bar is made of a Central steel plate 60mm wide and 10mm thick to which Copper plates 40mm wide by 5mm thick are Connected rigidly on each side. The length of the bar at normal temperature is 1m. If the temp is raised by 80°c, determine the stresses in each metal and the change in length. Take
$E_s = 200 GN/m^2$ ; $E_c = 100 GN/m^2$ ;
$\alpha_s = 12 \times 10^{-6} /°c$ ; $\alpha_c = 17 \times 10^{-6} /°c$.

Sol Given data ::

width of steel plate (w) = 60mm
Thickness of steel Plate (t) = 10mm
width of Cu plate (w) = 40mm
Thickness of Copper plate (t) = 5mm
length of the bar (L) = 1m
Temperature raised (T) = 80°c




$E_s = 200 GN/m^2$
$E_c = 100 GN/m^2$
$\alpha_s = 12 \times 10^{-6} /°c$
$\alpha_c = 17 \times 10^{-6} /°c$.

For equilibrium of forces

$$P_s = P_{cu} + P_{cu} \Rightarrow 2P_{cu}$$

$$\sigma_s A_s = 2\, \sigma_{cu} \cdot A_{cu}$$

$$\sigma_s = \frac{2 \times 40 \times 5}{60 \times 10} \cdot \sigma_{cu} \Rightarrow 0.66\, \sigma_{cu} \quad \text{—— (1)}$$

we also know

$$e_c = e_s$$

$$\alpha_{cu} T l - \frac{\sigma_{cu}}{E_{cu}} = \alpha_s T l + \frac{\sigma_s}{E_s}$$

$$80 \times 17 \times 10^{-6} - \frac{\sigma_{cu}}{1 \times 10^5} = 12 \times 10^{-6} \times 80 + \frac{0.66\ \sigma_{cu}}{2 \times 10^5}$$

$$1.36 \times 10^{-3} - 9.6 \times 10^{-4} = 3.3 \times 10^{-6} \sigma_{cu} + 1 \times 10^{-5} \sigma_{cu}$$

$$4 \times 10^{-4} = 1.33 \times 10^{-5}\ \sigma_{cu}$$

$$\boxed{\sigma_{cu} = 30.0751\ N/mm^2}$$

$$\sigma_s = 0.66\ \sigma_{cu}$$

$$\boxed{\sigma_s = 19.84\ N/mm^2}$$

Chang in length in the Compound bar

$$e_c = \frac{\delta l}{l} = \alpha_{cu} T l - \frac{\sigma_{cu}}{E_{cu}} \times l$$

$$= 17 \times 10^{-6} \times 80 \times 1000 - \frac{19.84}{1 \times 10^5} \times 1000$$

$$= 1.36 - 0.1984$$

$$\boxed{\delta l = 1.16\ mm.}$$

Q) Determine the value of young's modulus and poisson's ratio of a metallic bar of length 30cm, breadth 4cm and depth 4cm when the bar is subjected to an axial compressive load of 400kN. The decrease in length is given as 0.075 cm & increase in breadth is 0.003 cm.

Sol Given data

length of the bar (L) = 30cm = 300mm

breadth of the bar (b) = 4 cm = 40 mm

depth of the bar (d) = 4cm = 40mm

Axial compressive load (P) = 400kN = $4 \times 10^5$ N

change in length ($\delta l$) = 0.075cm = 0.75 mm.

change in breadth ($\delta b$) = 0.003cm = 0.03mm

we know

$$\text{longitudinal strain} = \frac{\delta l}{l} = \frac{0.75}{300} = 2.5 \times 10^{-3}$$

$$\text{Lateral strain} = \frac{\delta b}{b} = \frac{0.03}{40} = 7.5 \times 10^{-4}$$

$$\text{poisson's ratio} = \frac{\text{Lateral strain}}{\text{longitudinal strain}} = \frac{7.5 \times 10^{-4}}{2.5 \times 10^{-3}} = 0.3$$

we also know

$$\text{Longitudinal strain} = \frac{\sigma}{E} \Rightarrow E = \frac{\sigma}{\text{Longitudinal strain}}$$

$$E = \frac{P}{A \times e_1} = \frac{4 \times 10^5}{40 \times 40 \times 2.5 \times 10^{-3}} = \frac{\cancel{4} \times 10^5}{\cancel{4}}$$

$$\boxed{E = 1 \times 10^5 \ N/mm^2}$$

9) Determine the changes in length, breadth and thickness of a steel bar which is 4m long, 30mm wide and 20mm thick and it is subjected to an axial pull of 30KN in the direction of its length. Take $E = 2 \times 10^5 \, N/mm^2$ and poisson's ratio = 0.3.

Sol Given data

Length of the bar $(l) = 4m = 4000 \, mm$

width " " " $(w) = 30mm$

Thickness " " " $(t) = 20mm$

Axial pull $(P) = 30KN$

$E = 2 \times 10^5 \, N/mm^2$

$\mu = 0.3$

we know

$$\text{Poisson's ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Longitudinal strain } (e) = \frac{\sigma}{E} = \frac{P}{AE} = \frac{30 \times 10^3}{30 \times 20 \times 2 \times 10^5}$$

$$\frac{\delta l}{l} = \frac{30000}{12 \times 10^7} = 2.5 \times 10^{-4}$$

$$\delta l = 2.5 \times 10^{-4} \times 4000$$

$$\boxed{\delta l = 1mm}$$

~~Long~~ Lateral strain $= \mu \times$ Longitudinal strain

$= 0.3 \times 2.5 \times 10^{-4}$

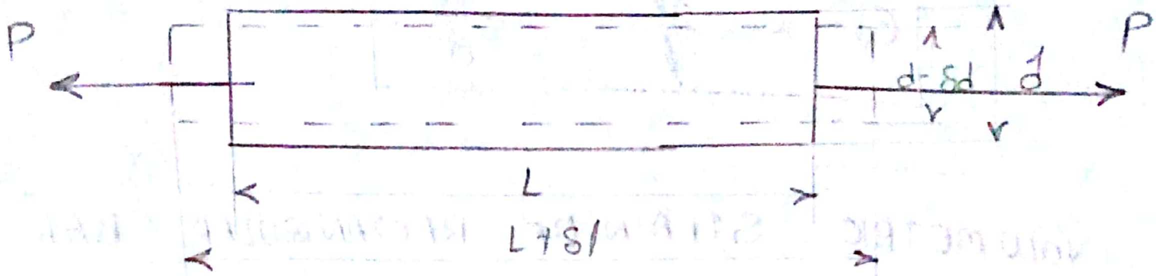
$\frac{\delta d}{d} = 7.5 \times 10^{-5}$

$$\boxed{\delta d = 2.25 \times 10^{-3} mm}$$

$\frac{\delta t}{t} = 7.5 \times 10^{-5} \Rightarrow \delta t = 7.5 \times 10^{-5} \times 20 \Rightarrow \boxed{\delta t = 1.5 \times 10^{-3} mm}$

## VOLUMETRIC STRAIN OF A CYLINDRICAL ROD:-

Consider a circular rod of diameter 'd' and length 'l' is subjected to axial tensile load.




we know volumetric strain ($e_v$) = $\frac{\delta V}{V}$ = $\frac{\text{Change in volu}}{\text{original volume}}$

change in volume = Final volume − Initial volume

Final volume of rod = $\frac{\pi}{4}(d-\delta d)^2 \times (L+\delta l)$

$= \frac{\pi}{4}(d^2 + \delta d^2 - 2\cdot\delta d\cdot d)(L+\delta l)$

$= \frac{\pi}{4}[d^2 L + d^2\delta l + \delta d^2 l + \delta d^2 \delta l - 2\delta d\cdot d\cdot l - 2\delta d \cdot d \cdot \delta l]$

neglecting the products of small quantities

$V_f = \frac{\pi}{4}[d^2 l + d^2\delta l - 2\,\delta d\cdot d\cdot l]$

$V_i$ (Initial volume) = $\frac{\pi}{4}d^2 \times l$

$$e_v = \frac{\delta V}{V} = \frac{V_f - V_i}{V_i} = \frac{(\frac{\pi}{4}d^2 l + d^2\delta l - 2d\cdot\delta d\cdot l) - \frac{\pi}{4}d^2 l}{\frac{\pi}{4}d^2 l}$$

$$= \frac{\cancel{\frac{\pi}{4}d^2 l} + d^2\delta l - 2d\,\delta d\, l}{\frac{\pi}{4}d^2 l} - \frac{\cancel{\frac{\pi}{4}d^2 l}}{\frac{\pi}{4}d^2 l}$$

$$= \cancel{\frac{\pi}{4}}[\cancel{d^2 l} + d^2\cdot\delta l - 2d\cdot\delta d\cdot l - \cancel{d^2 l}]$$

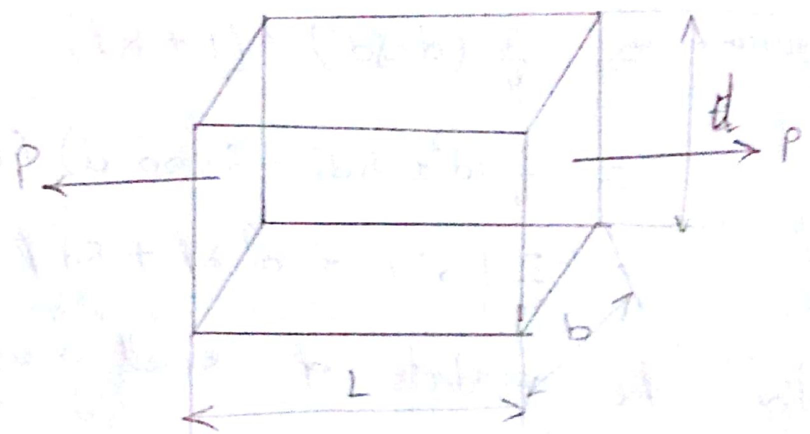
$$\frac{d^2\cdot\delta l - 2d\cdot\delta d\cdot l}{d^2\cdot l}$$

$$\frac{\cancel{d^2}\cdot\delta l}{\cancel{d^2}\, l} - \frac{2\cancel{d}\cdot\delta d\cdot\cancel{l}}{\cancel{d}\times d\times\cancel{l}}$$

$$\boxed{e_v = \frac{\delta l}{l} - \frac{2\,\delta d}{d}}$$

## VOLUMETRIC STRAIN OF RECTANGULAR BAR :-

Consider a rectangular bar of length $l$, width 'b' and thickness $d$. subjected to an axial tensile load as shown in below.




we know volumetric strain $(e_v) = \frac{\delta V}{V}$

Initial volume of Rectangular Bar $(V_i) = L\times b\times d$

Final volume of Rectangular bar $(V_f) = (L+\delta l)(b-\delta b)(d-\delta d)$

$= (L\cdot b - L\cdot\delta b + \delta l\cdot b - \delta l\cdot\delta b)(d-\delta d)$

$= L\cdot b\cdot d - L\cdot b\cdot\delta d - L\cdot\delta b\cdot d + L\cdot\delta b\,\delta d + \delta l\cdot b\cdot d - \delta l\, b\,\delta d$
$+ \delta l\cdot\delta b\cdot\delta d\,]$

neglecting the products of small quantities

$$= [L\cdot b\cdot d - L\cdot b\cdot \delta d - L\cdot \delta b\cdot d + \delta l\cdot b\cdot d]$$

Change in volume $(\delta V) = V_f - V_i$

$$= \cancel{L\cdot b\cdot d} - L\cdot b\cdot \delta d - L\cdot \delta b\cdot d + \delta l\cdot b\cdot d - \cancel{L\cdot b\cdot d}$$

$$= \delta l\cdot b\cdot d - L\cdot b\cdot \delta d - L\cdot \delta b\cdot d$$

$$e_v = \frac{\delta V}{V} = \frac{\delta l\cdot b\cdot d - L\cdot b\cdot \delta d - L\cdot \delta b\cdot d}{L\cdot b\cdot d}$$

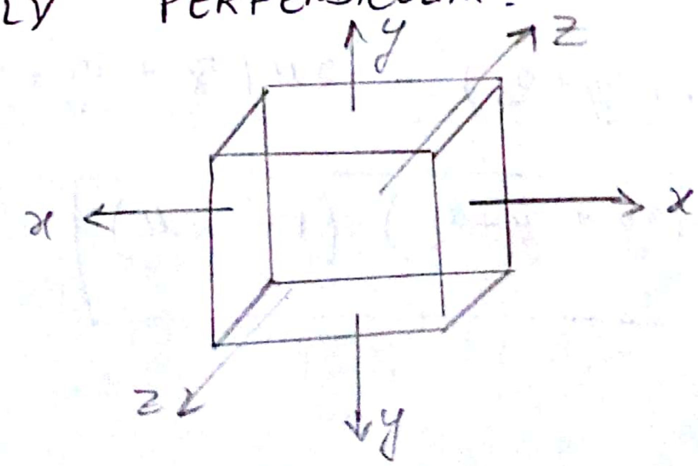
$$= \frac{\delta l\cdot \cancel{b\cdot d}}{L\cdot \cancel{b\cdot d}} - \frac{\cancel{L\cdot b}\cdot \delta d}{\cancel{L\cdot b}\cdot d} - \frac{\cancel{L}\cdot \delta b\cdot \cancel{d}}{\cancel{L}\cdot b\cdot \cancel{d}}$$

$$= \frac{\delta l}{l} - \frac{\delta d}{d} - \frac{\delta b}{b}$$

$$= e_l - e_{Lo} - e_{Lo}$$

$$\boxed{e_v = e_{Li} - 2e_{Lo}}$$

VOLUMETRIC STRAIN OF A RECTANGULAR BAR SUBJECTED TO THREE FORCES WHICH ARE MUTUALLY PERPENDICULAR :-




Volumetric strain $(e_v) = \frac{\delta V}{V} = e_x + e_y + e_z$

$$\mu = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

Latral strain = $-\mu$ Longitudinal strain

$e_x$ = strain in the direction of $x$.

$$e_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E}$$

$$e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$\frac{\delta V}{V} = e_x + e_y + e_z$$

$$\frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \mu \frac{\sigma_z}{E} + \frac{\sigma_y}{E} - \mu \frac{\sigma_y}{E} - \frac{\mu \sigma_z}{E} - \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E} - 2\mu \frac{\sigma_y}{E} - 2\mu \frac{\sigma_z}{E} - 2\mu \frac{\sigma_x}{E}$$

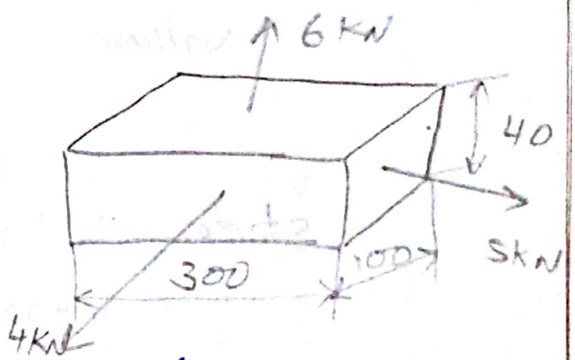
$$\frac{1}{E}(\sigma_x + \sigma_y + \sigma_z) - 2\mu(\sigma_x + \sigma_y + \sigma_z)$$

$$\boxed{\frac{\delta V}{V} = \frac{1}{E}(\sigma_y + \sigma_y + \sigma_z)(1 - 2\mu)}$$

9) A metallic bar 300mm x 100mm x 40mm is subjected to a force of 5 KN (tensile), 6 KN (tensile) and 4 KN (tensile) along x, y, and z directions respectively. Determine the change in volume of block. Take $E = 2 \times 10^5$ N/mm² & $\mu = 0.25$

Sol Given data

Volume of bar (V) $= 300 \times 100 \times 40$




we know

stress in x-direction $(\sigma_x) = \dfrac{\text{load in x-direction}}{\text{Area of } xy}$

$$= \frac{5000}{100 \times 40} = 1.25 \text{ N/mm}^2$$

llly

$$\sigma_y = \frac{6000}{300 \times 100} = 0.2 \text{ N/mm}^2$$

$$\sigma_z = \frac{4000}{300 \times 40} = 0.3333 \text{ N/mm}^2$$

$$e_v = \frac{\delta V}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$

$$\frac{\delta V}{300 \times 100 \times 40} = \frac{1}{2 \times 10^5}\left[(1.25 + 0.2 + 0.33)(1 - 2 \times 0.25)\right]$$

$$\frac{\delta V}{12 \times 10^5} = \frac{1}{2 \times 10^5}(1.7833)(0.5)$$

$$\delta V = 5.3499 \text{ mm}^3$$

9) A metallic bar 250 mm X 100 mm X 50 mm is loaded as shown in fig. Find the change in volume. Take $E = 2 \times 10^5$ N/mm$^2$ and poisson's ratio. (0.25)

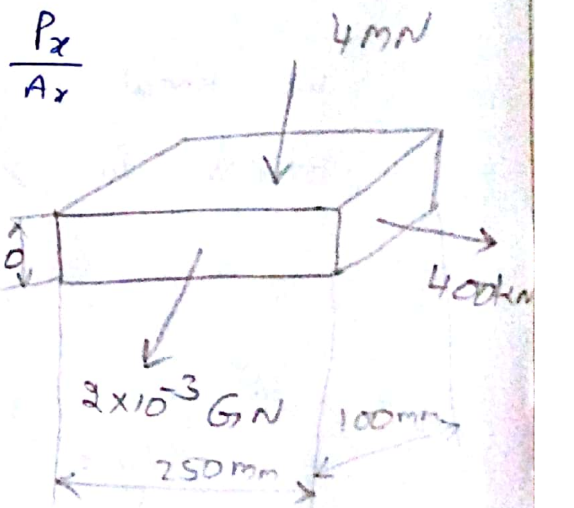
<u>Sol</u> Given data :-

volume of metallic bar (V) $= 250 \times 100 \times 50$

$= 125 \times 10^4$ mm$^3$

stress in x - direction $(\sigma_x) = \frac{P_x}{A_x}$

$= \frac{400 \times 10^3}{100 \times 50} = 80$ N/mm$^2$




$\sigma_y = \frac{4 \times 10^6}{250 \times 100} = 160$ N/mm$^2$

$\sigma_z = \frac{2 \times 10^{-3} \times 10^9}{250 \times 50} = 160$ N/mm$^2$

$e_v = \frac{\delta V}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$

$\frac{\delta V}{125 \times 10^4} = \frac{1}{2 \times 10^5}(80 + (-160) + 160)(1 - 2 \times 0.25)$

$\delta V = \frac{\overset{40}{\cancel{80}} \times 0.5}{\cancel{2} \times \cancel{10^5}} \times 125 \times \cancel{10^4}$

$\boxed{\delta V = 250 \text{ mm}^3}$

Q) A steel rod 5m long and 30mm in diameter is subjected to an axial tensile load of 50KN. Determine the change in length diameter and volume of the rod. Take $E = 2\times10^5\ N/mm^2$ and $\mu = 0.25$.

Sol Given data :-

Length of the rod (L) = 5m

Diameter of rod (D) = 30mm

Axial load (L) = 50KN

$E = 2\times10^5\ N/mm^2$

$\mu = 0.25$

we know

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E} = \frac{P}{AE} = \frac{50\times10^3}{\frac{\pi}{4}(30)^2\times2\times10^5}$$

$$\frac{\delta l}{l} = \frac{50000}{141371669.4} = 3.53\times10^{-4}$$

$$\delta l = 3.53\times10^{-4}\times5000$$

$$\boxed{\delta l = 1.76mm}$$

$$\text{Poisson's ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\mu\times e_l = e_{lateral}$$

$$\frac{\delta d}{d} = 0.25\times3.53\times10^{-4}$$

$$\delta d = 0.25\times3.53\times10^{-4}\times30$$

$$\boxed{\delta d = 2.6475\times10^{-3}}$$

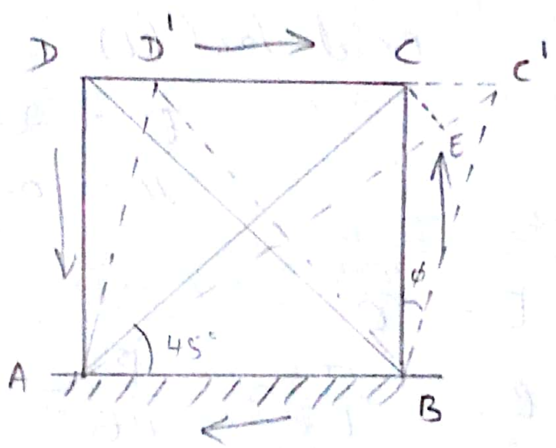
$$\frac{\delta V}{V} = \frac{\delta l}{l} - 2\frac{\delta d}{d}$$

$$= 3.53\times10^{-4} - 2(0.25 \times 3.53\times10^{-4})$$

$$\boxed{\delta V = 624.86\ mm^3}$$

## RELATIONSHIP BETWEEN ELASTIC CONSTANTS (E,G,K)

i) Relationship between E & G :-



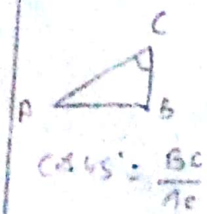

Consider a cube ABCD and apply ± shear force at DC face and fixed AB face. After applying the force the D changes to D' & C to C'.

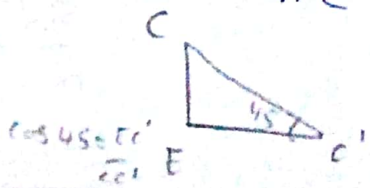
Draw a ⊥lar line on a new position i.e AC'.

Since the angle of CAE is very small, AC = AE.

We know strain in AC $= \frac{\delta l}{l} = \frac{AC' - AC}{AC}$

$$= \frac{EC'}{AC} = \frac{CC' \cos 45}{\frac{BC}{\cos 45^\circ}}$$



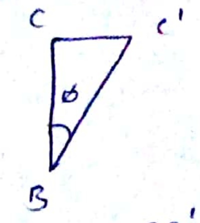



$$= \frac{CC'}{BC} \cos^2 45^\circ$$

$$= \phi \cdot \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \cdot \phi$$

$$\boxed{e = \frac{1}{2}\phi}$$

∴



$\tan\phi = \frac{CC'}{BC}$

$$e = \frac{\tau}{E}(1+\mu)$$

$$\phi = \frac{\tau}{G}$$

$$\frac{\tau}{E}(1+\mu) = \frac{1}{2} \cdot \frac{\tau}{G}$$

$$E = \frac{2G \cdot \tau}{\tau}(1+\mu)$$

$$\boxed{E = 2G(1+\mu)}$$

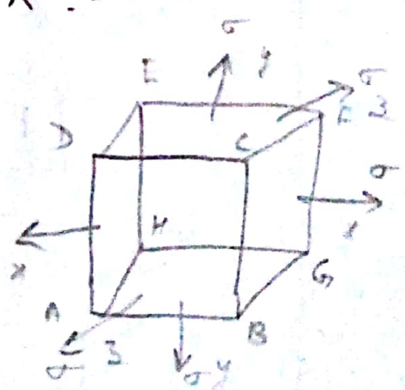
∴ Linear strain of diagonal AC = strain produced by tensile force along AC + strain produced by Comp. forces along BD

$$= \frac{\tau}{E} + \mu\left(\frac{\tau}{E}\right) \qquad \frac{\tau}{E} - \left(-\mu\frac{\tau}{E}\right)$$

$$= \frac{\tau}{E}(1+\mu)$$

ii) RELATIONSHIP BETWEEN E AND K :-

we know



volumetric Strain $(e_v) = \frac{\delta V}{V}$

Volume of Cube $= l^3$

Differentiating the above equation with respect to $l$

$$\frac{d}{dl}(V) = \frac{d}{dl}(l^3)$$

$$\frac{dV}{dl} = 3 \cdot l^2$$

$$dv = 3l^2 \cdot dl$$

$$\frac{dv}{V} = \frac{3\cancel{l^2}dl}{\cancel{l^3}} = 3\frac{dl}{l}$$

$$\boxed{\frac{dv}{V} = 3\frac{dl}{l}} \quad \text{--- (1)}$$

Total strain of ABCD face is.

$$\frac{\delta l}{l} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\frac{\delta l}{l} = \frac{\sigma}{E}(1 - 2\mu) \quad \text{--- (2)}$$

substitute eqn (2) in eqn (1)

$$\frac{dv}{V} = 3\frac{\sigma}{E}(1-2\mu)$$

$$e_v = 3\frac{\sigma}{E}(1-2\mu)$$

$$E = 3\frac{\sigma}{e_v}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

iii) Relationship between E, G and K:-

we know

$$E = 2G(1+\mu) \quad — ①$$

$$E = 3K(1-2\mu) \quad — ②$$

From eqn ①

$$\frac{E}{2G} = (1+\mu)$$

From eqn ②

$$\frac{E}{3K} = (1-2\mu)$$

multiply eqn ① by 2.

$$\frac{E}{2G} \times 2 = 2(1+\mu) \quad —$$

$$\frac{E}{G} = 2+2\mu \quad — ③$$

Add eqn ③ & ②

$$\frac{E}{G} + \frac{E}{3K} = 2+\cancel{2\mu}+1-\cancel{2\mu}$$

$$\frac{3KE+GE}{3KG} = 3$$

$$E(3K+G) = 9KG$$

$$\boxed{E = \frac{9KG}{3K+G}}$$

8) Determine the poisson's ratio and bulk modulus of a material, for which young's modulus is $1.2 \times 10^5$ N/mm² and modulus of rigidity $4.8 \times 10^4$ N/mm².

Sol Given data :-

young's modulus (E) $= 1.2 \times 10^5$ N/mm²

modulus of rigidity (G) $= 4.8 \times 10^4$ N/mm²

we know $E = 2G(1+\mu)$

$$1.2 \times 10^5 = 2 \times 4.8 \times 10^4 (1+\mu)$$

$$120000 = 96000 + 96000\mu$$

$$120000 - 96000 = 96000\mu$$

$$\mu = \frac{24000}{96000} = 0.25$$

$$\boxed{\mu = 0.25}$$

we Also know

$$E = 3K(1-2\mu)$$

$$1.2 \times 10^5 = 3K(1 - 2 \times 0.25)$$

$$K = \frac{1.2 \times 10^5}{3 - 1.5} = \frac{120000}{1.5}$$

$$\boxed{K = 8 \times 10^4 \text{ N/mm}^2}$$

9) A bar of cross-section 8 mm x 8 mm is subjected to an axial pull of 7000N. The lateral dimensions of the bar is found to be changed 7.9985 mm x 7.9985 mm. If the modulus of rigidity of the material is $0.8 \times 10^5 N/mm^2$ determine the poissons ratio and modulus of elasticity.

Sol Given data

Cross sectional Area (A) = 8 mm x 8 mm

= $64 mm^2$

Axial pull (P) = 7000 N

Lateral dimensions = 7.9985 x 7.9985

modulus of rigidity (G) = $0.8 \times 10^5 N/mm^2$.

We know

$$\text{poisson's ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Longitudinal strain } (e) = \frac{\sigma}{E} = \frac{7000}{8 \times 8 \times E}$$

$$e_L = \frac{109.375}{E}$$

$$\text{Lateral strain } (e_{Lateral}) = \frac{\delta l}{l} = \frac{8 - 7.9985}{8}$$

$$= \frac{1.5 \times 10^{-3}}{8}$$

$$\boxed{e_{Lateral} = 1.875 \times 10^{-4}}$$

$$\mu = \frac{1.857 \times 10^{-4}}{\frac{109.375}{E}}$$

$$\mu = 1.71428 \times 10^{-6} E$$

$$E = 2G(1+\mu)$$

$$E = 0.8 \times 10^{5} \times 2 \left(1 + 1.71428 \times 10^{-6} E\right)$$

$$E = (160000 + 0.2742 E)$$

$$(E - 0.2742E) = 160000$$

$$E(0.7258) = 160000$$

$$\boxed{E = 2.2 \times 10^{5}\ N/mm^2}$$

$$\mu = 1.71428 \times 10^{-6} \times 2.2 \times 10^{5}$$

$$\boxed{\mu = 0.37}$$

2) A steel plate 120mm long and 40mm wide is subjected to a tensile stress of 50 N/mm² in x direction and Compressive stress in y direction of 80 N/mm². If $E = 2.07 \times 10^5$ N/mm² & $\mu = 0.3$. calculate the change in length and width of plate.

Sol

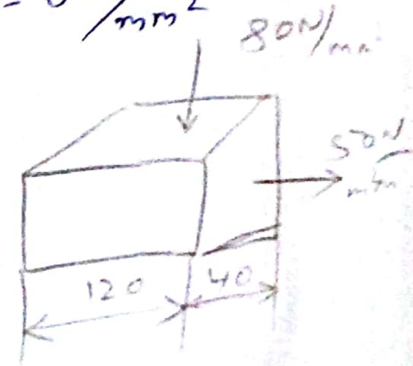
Given data :-

length of the plate $(l)$ = 120mm

width of the plate $(b)$ = 40mm

Tensile stress $(\sigma_t) = 50\,N/mm^2$

Compressive stress $(\sigma_c) = 80\,N/mm^2$




$$\boxed{e = \frac{\delta l}{l}}$$

But here two forces are acting on a body then

$$e = \frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$= \frac{50}{2.07\times10^5} - 0.3\left(\frac{50}{2.07\times10^5}\right)$$

$$\cancel{e_x = 2.415458.93\ 7.24\times10^{-5}}$$

$$e = 3.14\times10^{-4}$$

$$\frac{\delta l}{l} = 3.4\times10^{-4}\times120$$

$$\boxed{\delta l = 0.037mm}$$

$$e_y = -\frac{\sigma_y}{E} - \mu\frac{\sigma}{E}$$

$$= -\frac{80}{2.07\times10^5} - \frac{0.3\times50}{2.07\times10^5}$$

$$= 0.458\times10^{-3}$$

$$e_y = \frac{\delta b}{b} = 0.458\times10^{-3}\times40$$

$$\boxed{\delta b = 0.018\,mm}$$

# PART- B

Q1) A steel tube of 30mm external diameter and 20mm internal diameter encloses a Copper rod of 15mm diameter to which it is rigidly Joined at each end. If at a temperature of 10°c there is no longitudinal stress, Calculate the stresses in the rod and tube when the temp is raised to 200°c. Take E for steel and Copper as $2.1 \times 10^5$ N/mm$^2$ and $1 \times 10^5$ N/mm$^2$ respectively. The value of coefficient of linear expansion for steel and Copper is given as $11 \times 10^{-6}$ per °c and $18 \times 10^{-6}$ per °c respectively.

Sol Given data:-

steel tube external Dia $(D_s)$ = 30mm

steel tube internal Dia $(d_s)$ = 20mm

Copper rod diameter $(d_c)$ = 15mm

Initial temperature $(T_i)$ = 10°c

Final temperature $(T_f)$ = 200°c

Young's modulus for Copper $(E_c)$ = $1 \times 10^5$ N/mm$^2$

Young's modulus for steel $(E_s)$ = $2.1 \times 10^5$ N/mm$^2$

Coefficient of linear expansion for steel $(\alpha_s)$ = $11 \times 10^{-6}$/

Coefficient of linear expansion for Copper $(\alpha_c)$ = $18 \times 10^{-6}$/

we know $\quad$ Stress $(\sigma) = \frac{P}{A}$

$$P_s = P_c$$

$$\sigma_s A_s = P_c A_c$$

$$\sigma_s \left[\frac{\pi}{4}(D^2 - d^2)\right] = \sigma_c \frac{\pi}{4}(d)^2$$

$$\sigma_s \left[\frac{\pi}{4}(30^2 - 20^2)\right] = \sigma_c \frac{\pi}{4}(15)^2$$

$$\sigma_s (392.69) = \sigma_c (176.71)$$

$$\boxed{\sigma_s = 0.45\, \sigma_c} \quad \text{--- (1)}$$

we Also know in Case of Composite bars

$$e_s = e_c \qquad \boxed{\frac{\sigma}{e} = E}$$

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

$$11 \times 10^{-6} (200 - 10) + \frac{0.45 \sigma_c}{2.1 \times 10^5} = 18 \times 10^{-6} (200 - 10) - \frac{\sigma_c}{1 \times 10^5}$$

$$2.09 \times 10^{-3} + 2.1428 \times 10^{-6} \sigma_c = 3.42 \times 10^{-3} - 1 \times 10^{-5} \sigma_c$$

$$1.21428 \times 10^{-5} \sigma_c = 1.33 \times 10^{-3}$$

$$\boxed{\sigma_c = 109.52 \text{ N/mm}^2}$$

$$\sigma_s = 0.45\, \sigma_c$$

$$\boxed{\sigma_s = 49.28 \text{ N/mm}^2}$$